NEAT 2024

Bari, 19–22 November 2024

Program and abstracts

Last updated: Tuesday 19th November, 2024

Giuseppe Saccomandi:
A Visionary in Nonlinear Mechanics

NEAT2024 ORGANIZING COMMITTEE

Giuseppe Saccomandi's distinguished scientific journey began in the late 1980s when he had the privilege of learning from some of Italy's most eminent figures in Mathematical Physics, including G. Grioli, A. Belleni-Morante, and P. Benvenuti. His participation in the 1991 international conference honoring A. Signorini proved pivotal, as it introduced him to renowned experts in continuum mechanics such as P. Naghdi, T. Spencer, M. Hayes – his principal mentor – and other luminaries like G. Maugin and K. Dafermos. These encounters inspired his commitment to advancing the field of nonlinear elasticity.

Saccomandi's research stands out for its singular ability to blend mathematical rigor with an acute focus on cutting-edge physical problems. He has redefined continuum mechanics, introducing innovative multiscale modeling methods to describe complex phenomena like growth, phase transitions, and hierarchical material organization. His deep understanding of the historical context of mechanics, physics, and mathematics has guided his groundbreaking contributions. Notably, he co-authored Italy's most popular classical mechanics textbook for engineering students, reflecting his belief that "a good mathematical physicist uses only as much mathematics as necessary."

Over the years, Saccomandi has achieved exceptional results in fields such as soft material mechanics, nonlinear elasticity and viscoelasticity, biomechanics, and wave propagation. His multiscale approaches have forged explicit links between material properties at the microscopic scale and their macroscopic responses, particularly for biological and rubber-like materials. In biomechanics, his models have advanced our understanding of brain tissue mechanics and tissue damage diagnostics. His work on nonlinear wave propagation has yielded profound insights into elastodynamics, dispersive solids, and shear wave dynamics in anisotropic materials.

A thought leader in his field, Saccomandi has also made significant strides in fluid dynamics and experimental mechanics, addressing topics like piezoviscous fluids and refining classical approximations to resolve paradoxes. His theoretical advances include deriving universal relations via algebraic geometry and generalizing weak symmetry concepts in differential equations.

As Director of the National Group for Mathematical Physics (GNFM) at INdAM and former President of the International Society for the Interaction of Mechanics and Mathematics (ISIMM), Saccomandi has shaped the global scientific community. He has organized acclaimed courses and workshops and contributed to prestigious journals as editor. Despite his aversion to bureaucracy, his work has been generously funded, earning accolades such as the Ville de Paris Fellowship, the Carnot Star, and the Leverhulme Trust grant.

Above all, Saccomandi has been a tireless mentor to emerging researchers, many of whom are now gaining recognition in the international scientific arena. His legacy is one of innovation, mentorship, and an unwavering commitment to excellence in mathematical physics and mechanics.

With sincere esteem, gratitude and friendship,

D. Lacitignola, M. Ligabò, G. Napoli, G. Puglisi, I. Sgura, L. Vergori, R. Vitolo.

Stress and growth in cell aggregates

D. Ambrosi
Politecnico di Torino, Italia

Abstract: Cellular aggregates can be mechanically represented in an effective way as poroelastic solids that grow, where the growth law is coupled with the external loads and the active stress generated by the cells themselves. The mechanism of osmotic pressure offers the possibility to load cellular spheroids in an homogeneous fashion, thus providing the opportunity to check the predictions of mathematical models for a complex chemomechanical system while exploiting a simple geometrical setting. I will illustrate some experimental and numerical results that have at a good extent clarified the aforementioned interplays. Some recent experiments show that the active tension generated by the cells in spontaneous aggregates is not only affected by their stress, but also by their tensional history. When equipping the mathematical model with a simple memory rule, we are able to reproduce theoretically behaviors that cannot be captured by any traditional viscoelastic model.

Quantum fluids in graphene: a short overview

L. Barletti
Università degli Studi di Firenze, Italy

Abstract: The aim of the talk is to give a brief overview on the derivation of fluid-like equations describing the quantum dynamics of electrons in graphene. The main mathematical tool of such derivation is the classical Chapman-Enskog expansion together with the maximum entropy principle and the Wigner-Moyal calculus. Peculiar difficulties arise when applying these methods to graphene, mainly due to the conical intersection of the conduction and the valence bands and to the unboundedness from below of the latter. Such difficulties pose interesting mathematical challenges, especially in relation with the semiclassical expansion of the equations.

Two short stories about epithelia

M. Ben Amar
École Normale Supérieure, France

Abstract: TBA.

Necking of thin-walled cylinders and bifurcations of coated elastic disks

D. Bigoni
Università di Trento, Italia

Abstract: Necking localization under uniaxial tension is experimentally observed for a ductile thin-walled cylindrical tube, made up of soft polypropylene. Necking nucleates in multiple locations of the tube and spreads throughout it, involving also the occurrence of higher-order modes, evidencing trefoil and fourth-foiled shaped cross sections. No evidence in other ductile materials of such a complicated necking occurrence and growth were found for thin-walled cylinders. With the aim of theoretically modelling this phenomenon, as well as all other possible bifurcations, a two-dimensional formulation is introduced, in which only the mean surface of the tube is considered, paralleling the celebrated Flügge's treatment of cylindrical shells, subject to axial compression. That treatment is extended to include tension and a broad class of nonlinear-hyperelastic constitutive law for the material, which is also assumed to be incompressible [1]. This bifurcation problem is complemented with the akin bifurcation analysis of an elastic disk coated with a Cosserat isoperimetric constraint [2, 3]. The latter is treated as an elastic circular rod, either perfectly or partially bonded and is subjected to three different types of uniformly distributed radial loads (including hydrostatic pressure). As a particular case, a circular elastic rod is analyzed when subject to centrally-directed loads [4]. The presented results find applications in various fields, ranging from aerospace and automotive engineering to the vascular mechanobiology and morphogenesis of plants and fruits.

Acknowledgements. Financial support is gratefully acknowledged from the European Research Council (ERC), Grant agreement No. ERC-ADG-2021-101052956-BEYOND.

References:

[1] Springhetti, R., Rossetto, G., Bigoni, D. (2024) Necking of thin-walled cylinders via bifurcation of incompressible nonlinear elastic solids. Soft Matter, 20: 5703-5714.

[2] Gaibotti, M., Mogilevskaya, S.G., Piccolroaz, A., Bigoni D. (2024). Bifurcations of an elastic disc coated with an elastic inextensible rod. Proceedings of the Royal Society A, 480: 20230491.

[3] Gaibotti, M., Mogilevskaya, S.G., Piccolroaz, A., Bigoni D. (2024). Effects of prestress in the coating of an elastic disk. International Journal of Solids and Structures, 295: 112796.

[4] Gaibotti, M., Bigoni D., Cutolo, A., Fraldi, M, Piccolroaz, A., (2024) Effects of different loading on the bifurcation of annular elastic rods: theory vs. experiments. International Journal of Non-Linear Mechanics, 165: 104820.

Asymptotic behaviour of throughflow solutions in the class of Jeffreys' fluids

F. Capone
Università degli Studi di Napoli Federico II, Italy

Abstract: The present talk aims at providing a consistent model for non-isothermal viscoelastic fluid motion and at investigating the stability of a vertical constant throughflow in the fluid when it is saturating a horizontal layer heated from below.

In the context of a viscoelastic fluids[1], the material response can be described by a spring-dashpot model. Depending on the configuration and arrangement of springs and dashposts, the behaviour of a viscoelastic fluid is more or less elastic.

In the present talk, we consider a so-called *Jeffreys model*[2] (where spring and dashpot are in parallel and connected with another dashpot in series). In a Jeffreys model, the displacement of the elements in parallel is the same and it has to be summed with displacement of the second dashpot. As a result, in Jeffreys element the deformation is retarded by the presence of a dashpot in parallel with the spring. This model is indeed useful to describe fluids behavior.

Afterwords, the stability of a vertical constant throughflow[3] in a viscoelastic fluid of Jeffreys type, saturating a horizontal layer heated from below, is analyzed. Planes delimiting the layer are assumed isothermal, rigid and permeable. Stability analyses are performed and analytical and numerical results are reported.

References:

[1] D.D. Joseph. *Fluid dynamics of viscoelastic liquids*, Springer, 1990.

[2] I. Perez-Reyes, and L.A. Davalos-Orozco. *Effect of thermal conductivity and thickness of the walls on the natural convection in a horizontal viscoelastic Jeffreys fluid layer*, Journal of Heat Transfer, 2018.

[3] D. Nield, *Throughflow effects in the Rayleigh-Benard convective instability problem*, Journal of Fluid Mechanics, 1987.

Crack Propagation in Elastomeric Membranes: Coupling Between Finite Strain Elasticity and Rate-Dependent Effects

J. Ciambella
Università di Roma "La Sapienza", Italia

Abstract: Understanding crack propagation mechanisms in thin elastomeric membranes is crucial for various engineering applications, yet accurately modeling their complex behavior poses significant challenges due to the inherent material nonlinearity. In this contribution, we present a novel viscoelastic constitutive model specifically tailored to capture the time-dependent response of elastomers subjected to mechanical loading and damage, with particular emphasis on finite strain effects. Developed by following the classical approach to

phase field fracture [1], the model incorporates both short and long-term nonlinear elastic energy densities [2], as well as dissipation mechanisms [4], that are attenuated by damage through distinct degradation functions.

Our primary objective is to accurately predict crack propagation velocities in thin viscoelastic membranes, accounting for the geometric and material nonlinearities that characterize their behavior, a problem tested in several recent papers in the literature [3, 7, 5]. Energy dissipation around a propagating crack is the primary mechanism for the enhanced fracture toughness in elastomers. Such mechanism is spatially non-uniform and is highly coupled to the crack propagation process due to both the history-dependent nature of viscoelasticity and the large deformations involved. It is mainly driven by two processes: one intimately associated with viscoelasticity, which is controlled by the breaking and healing kinetics of ionic bonds as well as the degree of chemical crosslinking in the bulk material, and another occurring in the fracture process zone where the polymer chains experience extreme stretches, leading to significant strain-stiffening before their ultimate rupture.

By coupling finite strain elasticity, viscoelasticity, and damage, our model allows us to better highlight the combined role of material nonlinearity and dissipative mechanisms in crack propagation speeds. To validate the model, we conduct numerical simulations of a range of mechanical tests, including 1D and 2D simulations, that showcase the model's ability to capture the interplay between geometric nonlinearity, viscoelasticity, and crack propagation.

Joint work with G. Lancioni.

References:

[1] J. Ciambella, G. Lancioni, and N. Stortini. An Ogden-like formulation incorporating phase-field fracture in elastomers: from brittle to pseudo-ductile failures. *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.*, 380(2234), oct 2022.

[2] J Ciambella, G Lucci, and P Nardinocchi. Anisotropic evolution of viscous strain in soft biological materials. *Mech. Mater.*, 192:104976, may 2024.

[3] T. Corre, M. Coret, E. Verron, B. Leblé, and F. Le Lay. Experimental full field analysis for dynamic fracture of elastomer membranes. *Int. J. Fract.*, 224(1):83–100, 2020.

[4] F. D. Fischer, J. Svoboda, and H. Petryk. Thermodynamic extremal principles for irreversible processes in materials science. *Acta Mater.*, 67(153):1–20, 2014.

[5] Y Qi, X Li, S P Venkata, X Yang, T L Sun, C-y Hui, J P Gong, and R Long. Mapping deformation and dissipation during fracture of soft viscoelastic solid. *J. Mech. Phys. Solids*, 186(March):105595, may 2024.

[6] S Sadik and A Yavari. Nonlinear anisotropic viscoelasticity. *J. Mech. Phys. Solids*, page 105461, oct 2023.

[7] J Wang, B Zhu, C Y Hui, and A T. Zehnder. Delayed fracture caused by time-dependent damage in PDMS. *J. Mech. Phys. Solids*, 181(September):105459, 2023.

Optimal surface clothing with elastic nets

P. Ciarletta
Politecnico di Milano, Italia

Abstract: The clothing problem aims at identifying the shape of a planar fabric for covering a target surface in the three-dimensional space. It poses significant challenges in various applications, ranging from fashion industry to digital manufacturing.

Here, I will introduce a novel inverse design approach to the elastic clothing problem that is formulated as a constrained optimization problem. We assume that the textile behaves as an orthotropic, nonlinear elastic surface with fibers distributed along its warp and weft threads, and we enforce mechanical equilibrium as a variational problem. The target surface is frictionless, except at its boundary where the textile is pinned, imposing a unilateral obstacle condition for the reactive forces at the target surface. The constrained optimization problem also accounts for an elongation condition of the warp and weft fibers, possibly with bounded shearing angle. We numerically solve the resulting constrained optimization problem by means of a gradient descent algorithm. The numerical results are first validated against known clothing solutions for Chebyshev nets, taking the limit of inextensible fibers. We later unravel the interplay between thread and shear stiffness for driving the optimal cloth shape covering the hemisphere and the hemicatenoid. We show how the metric of these target surfaces strongly affects the resulting distribution of the reaction forces. When considering the limit of covering the full sphere, we show how clothing with elastic nets allows to avoid the onset of singularities in the corresponding Chebyshev net, by developing corners at the cloth boundary.

Joint work with D. Andrini and M. Magri, funded by MUR grants Dipartimento di Eccellenza 2023-2027, Italy and PRIN 2020 2020F3NCPX, Italy, and from European Commission through FSE REACT-EU funds, Italy, PON Ricerca e Innovazione, Italy.

Paradigms of deformable continua undergoing disarrangements

L. Deseri
Università degli Studi di Trento, Italia

Abstract: The multiscale mechanics of continua undergoing disarrangements have attracted considerable attention in the scientific community. While nonlinear field theories for elasticity in the presence of disarrangements are established, studies on their practical applications to real-world continuum mechanics remain limited. This presentation aims to address this gap by examining paradigmatic systems that undergo specific Structured Deformations. Analyzing their multiscale mechanical behavior, sometimes coupled with multiphysics interactions, can offer valuable insights into more systematic approaches for studying the onset of submacroscopic disarrangements. For example, depending on the specific material, this framework can be used to predict the behavior of solids at critical levels of damage or to analyze the overall response of materials, such as crystals or origami, that exhibit relatively small submacroscopic reconfigurations in comparison to the scale of their microstructure.

In vivo determination of pre-stress in skin

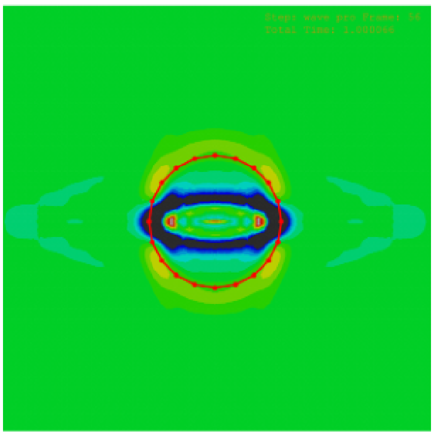
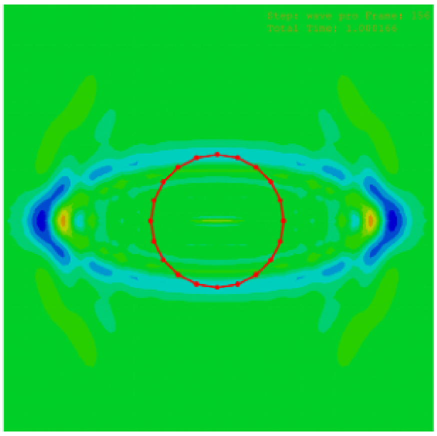
M. Destrade
University of Galway, Ireland

Abstract: Human skin is a complex material to test and model as its mechanical properties vary with several parameters, such as thickness, location, age, ethnicity, morphology, hydration, etc. Destructive testing of samples falls short, because harvesting skin dehydrates the sample, releases residual stress, and alters its response significantly. But measuring in skin vivo stress could be crucial for pre- and post-operative surgery planning, by providing safety limits.

Here, we develop and validate a method to quantify this stress with elastic wave measurements. We model the skin as an incompressible, anisotropic hyperelastic material with one family of fibres, with the principal pre-stress aligned with the fibre direction. The strain energy is the sum of the neo-Hookean potential (for the matrix) and *any* convex function of the first anisotropic invariant I_4. The analysis shows that the Rayleigh surface wave speeds give directly the in-plane stress difference through a simple formula, with an error of at most 9%.

We validate our formula first with finite element simulations using Abaqus. We create an in vivo stress by applying a pre-stretch and induce waves by an instantaneous point-load impulse. For simulations using the Holzapfel-Gasser-Ogden model, we find that the error is less than 5%. We also carried out experimental validation on synthetic tissue phantoms, and found good agreement with our simple formula.

This is joint work with Hannah Conroy-Broderick, Wenting Shu and Aisling Ní Annaidh (University College Dublin).

Analytical Results on Solitary Wave Propagation in Tensegrity Lattices

J. de Castro Motta
Università degli Studi di Salerno, Italia

Abstract: This research explores how solitary waves, both compressive and rarefaction, propagate within one-dimensional lattices composed of tensegrity-like springs and lumped masses on the quasi-continuum limit. The studied systems include interaction laws [1, 2, 3] between neighboring masses that simulate the structural response of tensegrity prisms. By applying a discrete-to-continuum transformation to the motion equations, the study identifies conditions under which compression and rarefaction solitary waves occur within a specific speed range. Detailed expressions are provided to define both the minimum and maximum propagation speeds, along with the solitary pulse's waveform. The model's capacity to forecast solitary wave propagation in a tensegrity mass-spring chain is validated through comparison with prior numerical findings [4, 5].

Joint work with A. Amendola and F. Fraternali.

References:

[1] J. de Castro Motta, K. Garanger, J. J. Rimoli. Propagation of compression solitary waves on tensegrity-like lattices made of truncated octahedrons. International Journal of Non-Linear Mechanics, (2024) 104716.

[2] A. Amendola. An analytic study on the properties of solitary waves traveling on tensegrity-like lattices, International Journal of Non-Linear Mechanics 148 (2023) 104264.

[3] de Castro Motta, J., Fraternali, F., Saccomandi, G. Rarefaction pulses on tensegrity lattices are just sech^2-solitary (dark) waves. Meccanica (2024). DOI: 10.1007/s11012-024-01853-8.

[4] F. Fraternali, G. Carpentieri, A. Amendola, R. E. Skelton, V. F. Nesterenko. Multiscale tunability of solitary wave dynamics in tensegrity metamaterials, Applied Physics Letters 105 (20) (2014).

[5] F. Fraternali, L. Senatore, C. Daraio. Solitary waves on tensegrity lattices, Journal of the Mechanics and Physics of Solids 60 (6) (2012) 1137-1144.

Self-Similar Solutions for the Heat Equation with a Positive non-Lipschitz Continuous, Semilinear Source Term

A. Farina
Università degli Studi di Firenze, Italy

Abstract: We investigate the existence of self-similar solutions for the parabolic equation $u_t = \Delta u + u^m H(u)$, with $0 \leq m < 1$ and H the Heaviside graph, coupled with the initial datum $u(\boldsymbol{x}, 0) = -c\left(|\boldsymbol{x}|^2\right)^{\frac{1}{1-m}}$, with $c > 0$. We analyze

two cases: the problem in R^n, $n > 1$, with $m = 0$ and the problem in R when $0 \le m < 1$. In the first case we show that there exist only two self-similar solutions changing sign, provided $0 < c < c_{cr}$, with c_{cr} obtained solving a specific algebraic equation depending on n. In the second case we prove that there exist at least two self-similar solutions of problem $u_t = u_{xx} + u^m H(u)$, $u(x,0) = -c\left(x^2\right)^{\frac{1}{1-m}}$, changing sign and evolving region where $u > 0$. These solutions are of great interest. Indeed, on one hand they prove that the problem does not admit uniqueness and on the other they prove that a single point where $u(x,0) = 0$, for an initial datum which is otherwise negative, can generate a region where $u(x,t)$ is positive.

Modeling Spider Silk Supercontraction as a Hydration-Driven Solid-Solid Phase Transition

G. Florio
Politecnico di Bari, Italia

Abstract: Spider silks have attracted significant interest due to their exceptional mechanical properties, which include a unique combination of high strength, ultimate strain, and toughness. A notable characteristic of spider silk, still debated from both mechanical and functional viewpoints, is supercontraction–a pronounced contraction of up to half its original length when an unconstrained silk thread is exposed to a wet environment. We propose a predictive model for the hygro-thermo-mechanical behavior of spider silks, conceptualizing this phenomenon as a solid-solid phase transition driven by humidity. As wetting increases, the system undergoes a transition, at the network scale, from a hard dry state to a soft amorphous wet state. We model these states using a two-well free energy function dependent on molecular stretch, with transition energy modulated by humidity. Based on the methods of Statistical Mechanics, we deduce that supercontraction can be interpreted as a solid-solid phase transition. We elucidate the important role of thermal fluctuations. Our model quantitatively predicts the observed experimental behavior, capturing the temperature dependence of humidity-induced supercontraction effects and related cooperative properties.

Mechanics of living matter and auxeticity behind the scenes

M. Fraldi
Università degli Studi di Napoli Federico II, Italia

Abstract: Continuum Mechanics is playing a key role in describing how growth, remodeling and morphogenesis in living matter do all interact with stress and fluid flow at the macro-scale. This coupling is often crucial to unveil complex underlying mechanisms originating from cascades of events occurring at lower scales. Cell-cell competitive dynamics can be in fact helpfully coupled with elasticity and mass balance equations and projected at the continuum level in order to obtain faithful outcomes that incorporate chemo-mechanical feedback

and give growth as a result rather than a preassigned datum. Interestingly, we can select some paradigmatic examples of biological systems where auxeticity seems to emerge as a chief factor in tissue mechanobiology, suggesting new possibilities of taking advantage of this counterintuitive –and sometimes overused– mechanical property, which in natural materials and man-made structures might be also hidden in symmetrical structures and activated by instabilities or even exhibited in absence of porosity.

This is a joint work with contributions by A.R. Carotenuto, A. Cutolo, S. Palumbo, L. Deseri and N.M. Pugno.

Stability in the vaneless diffuser of a centrifugal compressor

L. Fusi
Università degli Studi di Firenze, Italy

Abstract: This work is concerned with the theoretical aspects of flow stability in a vaneless diffuser of a centrifugal compressor. Specifically, the appearance of self-excited oscillations, also referred to as rotating stall, is investigated considering a two-dimensional inviscid flow in an annulus. We consider a linear perturbation method, taking as basic flow the rotationally invariant velocity field whose radial and tangential components are inversely proportional to the radial coordinate. We show that such flow may become unstable to small two-dimensional perturbations provided that the ratio between the inlet tangential velocity and the radial one is sufficiently large. Such an instability is purely kinematical, i.e. it does not involve any boundary layer effects, contrary to the classical hypothesis which ascribes the instability to a peculiar boundary layers interaction.

Conditional symmetries, straight on and backwards

G. Gaeta
Università degli Studi di Milano, Italia

Abstract: Giuseppe Saccomandi, who we are celebrating here, contributed in several ways to the theory of symmetries of differential equations; one of his contributions concerned the use of conditional symmetries *à rebours*. In this talk I will review this theory and explain why it is relevant in applications, both when used straight on and when used backwards.

Elastic Chirality

A. Goriely
University of Oxford, UK

Abstract: As the title suggests, I will show how to define a notion of chirality for elastic materials.

Helicity in dispersive continuum mechanics

H. Gouin
Aix Marseille Université, France

Abstract: By dispersive models of fluid mechanics we are referring to the Euler-Lagrange equations for the Hamilton action functional where the internal energy depends on high order derivatives of unknowns.

The corresponding Euler-Lagrange equations include, in particular, the van der Waals-Korteweg model of capillary fluids, the model of fluids containing small gas bubbles and the model describing long free-surface gravity waves. We obtain new conservation laws generalizing the helicity conservation for classical barotropic fluids.

Joint work with S. Gavrilyuk

Volumetric growth mechanics: a biomechanical "excuse" to study nonholonomic systems

A. Grillo
Polit. Bari, Italia

Abstract: In this contribution, I would like to summarize the most important results of a study that deals with the mechanics of volumetric growth, interpreted through the formalism and methodologies of Analytical Mechanics [1].

Following a modeling approach that has become rather standard in theoretical biomechanics, the minimal level of description of a biological medium experiencing growth (and viewed as a simple material) requires three main agencies: a source of mass; the deformation of the medium; and the reorganization of the medium's internal structure. The latter one is represented by a tensor field, often denominated *growth tensor*. In several circumstances, the time rate of the growth tensor can be assumed to be partially known from the outset. In our work, we regard this item of information as a *constraint*, and we name it *growth constraint*. In the cases that are meaningful from the biomechanical point of view, the growth constraint is *nonholonomic* (see, e.g., [2] for a definition). Yet, for reasons that will be explained in the talk, we embed such a constraint in a variational theory of volumetric growth [1].

To accomplish this task, we review and expand a method by Llibre et al. [3], termed *"Modified Vakonomic Method"* (MVM). This method was formulated for discrete mechanical systems with the intent of healing the inconsistencies affecting Kozlov's Vakonomic Method in the solution of some benchmark problems. While adapting the MVM to the continuous framework of growth mechanics, we highlight some fundamental differences between our formulation of this procedure [1] and the one presented in [3]. In particular, we show that these differences descend from the physical requirement that the dynamics of growth obtained variationally be the same as those determined, e.g., by means of the Principle of Virtual Work [4].

Joint work with A. Pastore, S. Di Stefano.

References:

[1] Grillo, A., Pastore, A., Di Stefano, S.: "An approach to growth mechanics based on the analytical mechanics of nonholonomic systems", *J. Elasticity* (2024). To appear.

[2] Neimark, J., Fufaev, N.A.: *Dynamics of Nonholonomic Systems*. Am. Math. Soc., Rhode Island (1972).

[3] Llibre, J., Ramírez, R., Sadovskaia, N.: "A new approach to the vakonomic mechanics", *Nonlinear Dynamics*, **87**, 2219-2247 (2014).

[4] DiCarlo, A., Quiligotti, S.: "Growth and balance", *Mech. Res. Commun.*, **29(6)**, 449-456 (2002).

A kinetic approach to action potential modelling on neural networks

M. Groppi
Università degli Studi di Parma, Italia

Abstract: In this talk a kinetic model capable to reproduce the action potential dynamics in a neural network representing a brain region is proposed. Two levels of description are coupled: in a single brain area, pairwise neuron interactions for the exchange of membrane potential are statistically described; among different areas, a graph description of the brain network topology is included. Macroscopic equations for the membrane potential and the recovery variable can be consistently deduced under suitable assumptions. Equilibria of the kinetic and macroscopic settings are determined and numerical simulations of the system dynamics are performed, with the aim of studying the influence of the network topology and heterogeneities on the membrane potential propagation and synchronization.

Undular bores and how to describe them

K. Khusnutdinova
Loughborough University, UK

Abstract: Undular bores, or dispersive shock waves, are non-stationary waves propagating as oscillatory transitions between two basic states, in which the oscillatory structure gradually expands and grows in amplitude with distance travelled. Such waves are ubiquitous in nature, and in this talk we will give a brief overview of longitudinal undular bores in impact tests, as well as the bores generated by natural and induced tensile fracture. We will then discuss modelling using the viscoelstic extended Korteweg - de Vries equation and useful estimates which can be obtained using the linear bore approximation. We will also overview comparisons between theoretical predictions and experimental data.

Coherent structures in an IntraGuild predation model with anti-predator behavior

M.C. Lombardo
Università degli Studi di Palermo, Italia

Abstract: This study introduces the following model with nonlinear cross-diffusion term, defined on the spatial domain $\Omega \subset \mathbb{R}$:

$$\begin{cases} \frac{\partial N_1}{\partial t} = D_1 \Delta N_1 + N_1 \left(B - a_{11} N_1 - a_{12} N_2 - a_{13} N_3 \right), & \text{on } \mathbb{R}_+ \times \Omega, \\ \frac{\partial N_2}{\partial t} = D_2 \Delta N_2 + D \nabla \cdot \left(N_2 \nabla N_3 \right) + N_2 \left(-M_2 + a_{21} N_1 - a_{23} N_3 \right), & \text{on } \mathbb{R}_+ \times \Omega, \\ \frac{\partial N_3}{\partial t} = D_3 \Delta N_3 + N_3 \left(-M_3 + a_{31} N_1 + a_{32} N_2 \right), & \text{on } \mathbb{R}_+ \times \Omega, \end{cases} \tag{1}$$

where $N_1(t, X), N_2(t, X), N_3(t, X)$, represent the densities of a basal resource, an intermediate consumer (the IGPrey) and an omnivorous predator (the IG-Predator), respectively. The model describes the dynamics of intraguild predation communities, where predators and prey also compete for shared resources within an ecological guild. It incorporates IntraGuild Prey exhibiting anti-predator behavior, dispersing along local gradients in predator density, while local dynamics are described by the Lotka-Volterra functional form [1].

We prove that the predator avoidance strategy described by cross-diffusion plays a crucial role for pattern formation in the reaction-diffusion system and we characterize the cross-diffusion-driven Turing bifurcation. Using the formalism of amplitude equations, we derive the asymptotic profiles of the stationary solutions, revealing that anti-predator behavior can explain segregation patterns between IntraGuild Prey and IntraGuild Predator observed in field studies. Through a combination of analytical and numerical tools, we demonstrate that the predator avoidance strategy serves as a mechanism that stabilizes coexistence states in Intraguild Predation communities beyond the conditions imposed by the corresponding spatially homogeneous model.

In the latter part of the presentation, we address the issue of travelling wave solutions to (1), with applications to invasion phenomena of the Atlantic blue crab in the Mediterranean region.

References:

[1] F. Farivar, G. Gambino, V. Giunta, M.C. Lombardo, M. Sammartino, *Intraguild predation communities with anti-predator behavior*, SIAM J.Appl.Dyn.Sys., (2024), *in press.*

The generalised hodograph method for non-diagonalisable integrable systems of hydrodynamic type

P. Lorenzoni
Università degli Studi Milano Bicocca, Italia

Abstract: The generalised hodograph method is the main tool to solve integrable systems of hydrodynamic type admitting Riemann invariants. In this talk, based in a joint work with Sara Perletti and Karoline van Gemst, I will explain how to extend the method to regular non-diagonalisable integrable systems of hydrodynamic type exploiting the relation between such systems and F-manifolds with compatible connections.

Persistent Use and Misuse of Worm Like Chain Models

J.H. Maddocks
EPFL, Switzerland

Abstract: I have not co-written an article with Giuseppe Saccomandi (at least not yet), but we have many common research areas of interest. One of these is Giuseppe's contributions to bringing the level of rigour customary in applied mathematics and mechanics to applications of models such as the Worm Like Chain, or WLC [1, 2], which is widely adopted in molecular biology and biochemistry, including the specific case of DNA [3]. The WLC model has one free stiffness parameter, typically called the persistence length, but no intrinsic application length scale *per se*. This makes the WLC a powerful tool in modelling experimental data at many scales, but has also lead to its misuse at length scales where the underlying hypotheses of the WLC are no longer close to being valid. For example, for DNA its persistence length is widely stated to be around 50 nm or 150 base pairs (or bp) with no significant dependence on base-pair sequence. However at the length scale of 10-100 bp or so the double helix centreline is now known to be very far from intrinsically straight, with biologically significant, and highly sequence-dependent, static bends. In this talk I will describe work of my then group [4] in developing a rigorous notion of *dynamic* persistence length, which when applied to DNA varies by up to 50% with sequence, and with a sequence-averaged value of well over 200 bp.

References:

[1] R.W. Ogden, G. Saccomandi, I. Sgura, "On worm-like chain models within the three-dimensional continuum mechanics framework" Proc Roy Soc A, 2006, **462**

[2] R.W. Ogden, G. Saccomandi, I. Sgura, "Computational aspects of Worm-Like-Chain interpolation formulas", Computers & Mathematics with Applications, 2007, **53**

[3] G Saccomandi, I Sgura, "The relevance of nonlinear stacking interactions in simple models of double-stranded DNA", Journal of the Royal Society Interface, 2006, **3**

[4] J. Mitchell, J, Glowacki, A.E. Grandchamp, R.S. Manning and J.H. Maddocks, "Sequence-Dependent Persistence Lengths of DNA," Journal of Chemical Theory and Computation, 2017, **13**, 10.1021/acs.jctc.6b00904

On the concept of self-interpenetration for tubular elastic bodies

A. Marzocchi
Università Cattolica del Sacro Cuore, Brescia, Italia

Abstract: After a brief review of the most famous conditions for non-interpenetration of continuum bodies, a proposal coming from the theory of knots is presented, together with some analytical results. More precisely, if γ is the midline curve with Serret-Frenet frame $\{\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}\}$, supposed to be regular, and $T_r[\gamma]$ is the tubular neighbourhood of γ, and if

$$M(s)(\theta) = \gamma(s) + r\cos\theta\boldsymbol{n}(s) + r\sin\theta\boldsymbol{b}(s), \theta \in [0, 2\pi]$$
$$P(\theta)(s) = \gamma(s) + r\cos\theta\boldsymbol{n}(s) + r\sin\theta\boldsymbol{b}(s), s \in [0, L]$$

define respectively the meridians and the parallels of $T_r[\gamma]$, then an additional repulsive energy of the form

$$E(T_r) = \int_{\mathrm{bd}\, T_r \times \mathrm{bd}\, T_r} \left(\frac{1}{|X-Y|^2} - \frac{1}{{d^*}^2(X,Y)} \right) dS\, dS =$$
$$= \int_0^L \int_0^L \int_0^{2\pi} \int_0^{2\pi} \left(\frac{1}{|p(r,s,\theta) - p(r,t,\varphi)|^2} - \frac{1}{{d^*}^2(p(r,s,\theta), p(r,t,\varphi))} \right) d\varphi\, d\theta\, dt\, ds$$

where p is the positioning of point with coordinates θ, φ on the section through $\gamma(s)$, $|X - Y|$ is the Euclidean distance

$$|X-Y|(s,\theta,t,\varphi) = |\gamma(s) - \gamma(t) + r(\cos\theta\boldsymbol{n}(s) - \cos\varphi\boldsymbol{n}(t)) + r(\sin\theta\boldsymbol{b}(s) - \sin\varphi\boldsymbol{b}(t))|$$

and ${d^*}^2(X,Y)$ is, assuming $s < t$ for simplicity,

$${d^*}^2(X,Y)(s,\theta,t,\varphi) = (l_{M(s)} + \hat{l}_P^{\boldsymbol{n},\boldsymbol{b}})^2 + \hat{l}^{\boldsymbol{t}}_{P(\theta)} \hat{l}^{\boldsymbol{t}}_{P(\varphi)}$$

where $l_{M(s)}$ is the length of the meridian through $\gamma(s)$, and

$$l^{\boldsymbol{t}}_{P(\theta)} = \int_s^t ||P'(\xi) \cdot \boldsymbol{t}(\xi)||\, d\xi, \qquad l^{\boldsymbol{n},\boldsymbol{b}}_{P(\theta)} = \int_s^t ||P'(\xi) \times \boldsymbol{t}(\xi)||\, d\xi$$

$P(\theta)$ being the parallel at angle θ, is capable to distinguish between configurations with or without self-contact, in the sense that it diverges if and only if the tubular neighbourhood presents self-contact or self-interpenetration.

A variational model for fracture and cavitation in almost incompressible materials

C. Maurini
Université Pierre et Marie Curie and CNRS, Paris, France

Abstract: Energetic approaches to brittle fracture model crack nucleation and propagation as a competition between the elastic energy stored in the material and the energy required to create the crack. This approach appears to break down for very stiff materials, where the elastic energy seems negligible compared to the energy required to nucleate a crack. An emblematic example is the case of nominally incompressible or almost incompressible materials subjected to dominant pressure loading. We propose a phase-field model for cavitation and fracture in almost incompressible materials that reconciles the energetic argument with the experimental results of the famous poker-chip test by Gent and Lindley on natural rubber. Our model is based on a macroscopic homogenized representation of the cavity formation mechanism as a nonlinear elastic phenomenon, coupled with a phase-field model of brittle fracture. Consistent with experimental observations, we argue that cavity nucleation and growth after the critical cavitation pressure lead to the presence of a phase with possible macroscopic volumetric deformations, allowing the storage of elastic energy that is released during the crack nucleation and propagation process. Our theoretical argument and numerical results demonstrate how variational models can capture the main features of the cavitation and fracture process in almost incompressible materials based solely on energy minimization, without the addition of any ad hoc criteria or driving forces.

Variations on a theme by Peppe Saccomandi

J.G. Murphy
Dublin, Ireland

Abstract: Universal relations for isotropic, homogeneous, non-linearly elastic materials are used to explore the consequences for the strain of assuming a given stress field, with a particular emphasis on the problem of simple shear. The classical semi-inverse approach of Rivlin for the canonical problems of non-linear elasticity is compared and contrasted with this new stress-based formulation.

Nonlinear elastic deformation in swelling gel-based long cylinders and thin discs

P. Nardinocchi
Università di Roma "La Sapienza", Italia

Abstract: Gels consist of polymer network and liquid; placed in a liquid bath, gels absorb liquid and swell. The simplest problem in mechanics of swelling is the free-swelling problem: a dried gel placed in a good solvent swells absorbing the solvent from the surrounding.

Mechano-diffusion theory predicts that a dry gel with volume V_d and shear modulus G_d swells to a volume $V_\infty = J_d\, V_d$ when immersed in a good solvent. The swelling ratio J_d only depends on material parameters and temperature, and gels with the same dry volume V_d will absorb the same solvent volume $V_s = (J_d - 1)\, V_d$.

Shape does not matter.

On the contrary, swelling dynamics is affected by a characteristic length, which is related to the body geometry. To stress the relationship between swelling-driven deformations and geometry, we consider a family of isovolumetric cylinders and discs of radius R_d and thickness H_d, having different aspect ratio $a_r = 2R_d/H_d$. Each element of the family swells to a same volume V_∞, absorbs the same amount of solvent V_s and keep its own swelling ratio unchanged. By

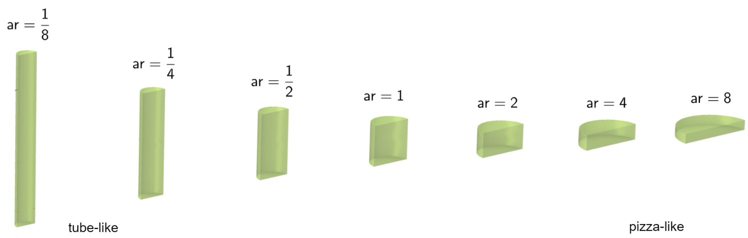


Figure 1: Half sections of isovolumetric cylinders and discs of aspect ratio $a_r = 2R_d/H_d$. From left to right $a_r = 1/8, 1/4, 1/2, 1, 2, 4, 8$.

solving a nonlinear mechano-diffusion problem, we show how shape emerges as a critical factor in transient dynamics, with aspect ratio dictating the system's response.

Continuous limit of a traction-unstable metastructure

A. Nobili
Università degli Studi di Modena e Reggio Emilia, Italy

Abstract: The continuous limit for a metamaterial composed of several unit cells which are unstable under traction is developed within the realm of elastodynamics. The resulting variational structure is crucially supplemented by a unilateral constraint, which turns the otherwise linear problem into a complementarity problem. Indeed, the absence of the unilateral constraint significantly alters the response of the system, notably missing the fundamental equilibrium state. We show that proper dealing with the unilaterally constrained Lagrangian leads to the reproduction at the macro-level of all the features exhibited by the microstructure. The Hamiltonian of the system is also discussed and plays a fundamental role in addressing the non-smooth transition between the equilibrium states.

Extended procedures for the exploitation of entropy inequality: the case of Korteweg fluids

F. Oliveri
Università degli Studi di Messina, Italia

Abstract: This talk deals with the exploitation of Clausius-Duhem inequality for a Korteweg fluid, *i.e.*, a fluid whose stress tensor is allowed to be dependent on first and second order gradients of mass density, There are two well known strategies for the exploitation of entropy principle, namely the Coleman-Noll procedure [1] and the Liu one [2]. For continua with nonlocal constitutive equations the classical procedures may fail unless a modification of the balance of energy (for instance including extra contributions like the interstitial working [3] modeling long range interactions), or of the entropy inequality by assuming an *a priori* entropy extra flux [4], is introduced.

Extended procedures, that use as constraints in the entropy inequality both the field equations and some of their gradient extensions up to the order of derivatives entering the state space, allow to determine the compatibility with the second law of thermodynamics without modifying the balance of energy. Remarkably, the procedure may produce a contribution to entropy flux besides the classical one without assuming it from the very beginning.

We illustrate the procedure and the results in the case of Korteweg fluids [5]. Remarkably, the constitutive equations are such that, considering the purely mechanical equilibrium configurations, the conditions established by Serrin, based on a result by Pucci [7], are satisfied. This implies that the equilibria at interface could not be restricted to simple geometric phase boundaries (spherical, cylindrical, or planar). In fact, the overdetermined system characterizing equilibria reduces to a single elliptic equation.

References:

[1] B.D. Coleman, W. Noll, Arch. Ration. Mech. Anal. **13** (1963), 167–178.

[2] I.-S. Liu, Arch. Ration. Mech. Anal. **46** (1972), 131–148.

[3] J.E. Dunn, J. Serrin, Arch. Ration. Mech. Anal. **88** (1985), 95–133.

[4] I. Müller, Arch. Ration. Mech. Anal. **26** (1967), 118–141.

[5] M. Gorgone, P. Rogolino, Phys. Fluids **33** (2021), 043107.

[6] J. Serrin, Q. Appl. Math. **41** (1983), 357–364.

[7] P. Pucci, Q. Appl. Math. **41** (1983), 365–367.

Multi-level biomechanical models for cell migration in dense fibrous environments

L. Preziosi
Politecnico di Torino, Italia

Abstract: Cell-extracellular matrix interaction and the mechanical properties of the cell nucleus have been demonstrated to play a fundamental role in cell movement across fibrous networks, micro-channels and therefore the microstructure of porous scaffolds. So, their study is important to understand both motion and growth in confined environments and also the spread of cancer metastases. This talk will merge the results of some continuum mechanics models and individual cell-based models that take into account of cell adhesion mechanics and nucleus mechanical properties to finally deduce a macroscopic model able describe the motion and growth in dense fibrous environments.

Virtual trapping of Saccomandi by a spider evolved with knotting ability

N.M. Pugno
Università degli Studi di Trento, Italia, and Queen Mary University of London, UK

Abstract: Prof. Giuseppe Saccomandi (Peppe) has fundamentally advanced mechanics in a variety of fields, incidentally including also spider silk [1]. That paper -written also with prof. Giuseppe Puglisi with whom I have the privilege of collaborating on this topic [2]- gave me inspiration for this talk in honor of Peppe. Spider silk is renowned for its exceptional mechanical properties, combining low density with high tensile strength and high extensibility and thus very high toughness modulus (t, i.e. dissipated energy per unit mass). However, the potential toughness of spider silk could be significantly enhanced if spiders evolved the -currently absent/undiscovered- ability to tie knots in their silk. This advancement would allow for a new level of gigantic toughness (T) revealing a today "hidden toughness", mimicking human engineering techniques and in particular a related proposal by the author used for realizing the world toughest fibers [3]. Indeed, knotting could provide additional energy dissipation via friction, enabling spiders to construct webs and traps with unprecedented efficiency. To quantify this scenario, the author calculates the gigantic toughness of 393 real spiders virtually assumed with evolving knot-making behaviors, showing toughness gain ($G = T/t$) of about one or two orders of magnitude [4]. A virtual (Figure 1) trapping of Peppe is thus discussed considering the emerging highest gigantic toughness ($Tmax$). As a gift for his 60 years anniversary, Peppe will be released by the spider thanks to a recently discovered efficient cutting mechanism [5].

References:

[1] D. De Tommasi, G. Puglisi, G. Saccomandi, Damage, Self-Healing, and Hysteresis in Spider Silks, BIOPHYSICAL JOURNAL, 2010, 98, 1941-1948.

Figure 2: Virtual trapping of a professor by a spider (created by current artificial intelligence, ChatGPT)

[2] V. Fazio, N. M. Pugno, G. Puglisi, "Water to the ropes": A predictive model for the supercontraction stress of spider silks, Extreme Mechanics Letters, 2023, 61, 102010

[3] N. M. Pugno, The "Egg of Columbus" for making the world's toughest fibres. PLoS ONE, 2014, 9, e93079.

[4] N. M. Pugno, Super-tough silk: the potential of knots in evolved spiders. Under review.

[5] G. Greco, D. Misseroni, F. Castellucci, N. G. Di Novo, N. M. Pugno, Functionally-Graded Serrated Fangs Allow Spiders to Mechanically Cut Silk, Carbon and Kevlar Fibers, Advanced Science, 2024, `https://onlinelibrary.wiley.com/doi/10.1002/advs.202406079`

Rethinking the development of constitutive relations

K. Rajagopal
Texas A& M University, USA

Abstract: After discussing the rationale and the need for rethinking the development of constitutive relations, both from philosophical and pragmatic viewpoints, to describe the response of both non-linear fluids and solids, I will discuss the development of a new class of constitutive relations and applications wherein they can be gainfully exploited. It will be shown that the new constitutive relations can explain phenomena that have hitherto defied adequate explanation.

Stress Relaxation and Viscous Energy in Nonlinear Viscoelasticity: A Rational Extended Thermodynamics Framework

T. Ruggeri
Università degli Studi di Bologna, Italia

Abstract: In this talk, we briefly review a recent hyperbolic model of nonlinear viscoelasticity based on the principles of Rational Extended Thermodynamics [1]. We then focus on analyzing uniaxial stress relaxation under constant strain, determining the viscous dissipated energy such that the stress decays over time as a sum of exponential terms (a Prony series) with varying relaxation times. Our results show that the derived viscous energy satisfies all model requirements, ensuring the system's symmetric hyperbolic nature and compliance with the dissipation principle. According to the model's assumptions, which requires that viscous energy depends only on the viscous stress, we derive the analytical form of the coefficients based on the initial deformation step. This approach allows us to predict viscous stress decay for any deformation jump, relying only on the fitting coefficients obtained from one experiment. We validated our approach by successfully reproducing experimental data from uniaxial relaxation tests on a woven Dacron fabric, commonly employed in aortic grafts. Part of this presentation is the subject of a paper in collaboration with M. Amabili and T. Arima [2].

References:

[1] Ruggeri, T., A nonlinear approach to viscoelasticity via rational extended thermodynamics, Int. J. Non-Linear Mech., 160, 104658 (2024).

[2] Amabili, M., Arima, T., Ruggeri, T. Stress Relaxation and Viscous Energy in Nonlinear Viscoelasticity: A Rational Extended Thermodynamics Framework, Submitted.

Well-Posedness of the Viscous 2D MHD Vorticity-Current Equations

M. Sammartino
Università degli Studi di Palermo, Italia

Abstract: We consider the two-dimensional viscous magneto-hydrodynamic equations in the whole plane. After briefly reviewing a recent short-time existence result for L^1 data, we investigate the global in-time well-posedness problem. In particular, we consider initial data that have the radial-energy decomposition form, i.e., the initial velocity field (and analogously the initial magnetic field) can be decomposed in two vector fields, where one has finite L^2 energy and the other one has radial symmetry. For these data, we can extend the solution globally in time. As a particular case, we investigate the vortex-patch configuration, where we can establish a zero-viscosity limit result, showing that the solutions converge to the ideal MHD solution.

Joint work with Maria Schonbek and Vincenzo Sciacca

Well-posedness for a reaction-diffusion-chemotaxis model of multiple sclerosis

V. Sciacca
Università degli Studi di Palermo, Italy

Abstract: We consider a reaction-diffusion-chemotaxis model of multiple sclerosis where the modulatory influence of cytokines on the activation rate of macrophages is incorporated.

The model is an extension of [Lombardo et al. (2017), *Journal of Mathematical Biology*, **75**, 373–417].

We establish the existence of a unique global solution to our proposed system when the activation rate exhibits linear growth with increasing cytokine levels.

Joint work with F. Gargano, M.C. Lombardo, R. Rizzo and M. Sammartino. *International Journal of Non-Linear Mechanics*, **161** (2024), 104672.

Small-on-Large Geometric Anelasticity

S. Souhayl
Aarhus University, Denmark

Abstract: The complexity of the equations of anelasticity leaves little hope for exact solutions to be found. A few can be found by semi-inverse methods assuming some symmetry-restrictive classes of deformations. As soon as this symmetry is broken, the governing equations start to be utterly complicated leaving no choice but for equally complicated numerical computations. In this work, a new theory of small-on-large geometric anelasticity is formulated by considering a small perturbation superposed on a finite eigenstrain distribution (due to some anelastic effect). In the geometric anelasticity framework, this corresponds to perturbing the material metric in the reference configuration. Such a perturbation of the referential geometry yields a small elastic deformation superposed on the finite deformation due to the original eigenstrain distribution. The governing equations are linear. Even in the case when one fails to find exact solutions, these are much easier to deal with numerically. If the perturbation lacks the symmetry of the original distribution, one hence generates new solutions for anelastic problems. In particular, this theory is used to find exact solutions for some non-symmetric distributions of screw dislocations in incompressible isotropic solids—see Fig. 3. Beyond generating new solutions for non-symmetric anelasticity problem, this small-on-large approach lays the foundational framework for examining the stability of eigenstrain distributions.

Joint work with Arash Yavari from Georgia Institute of Technology.

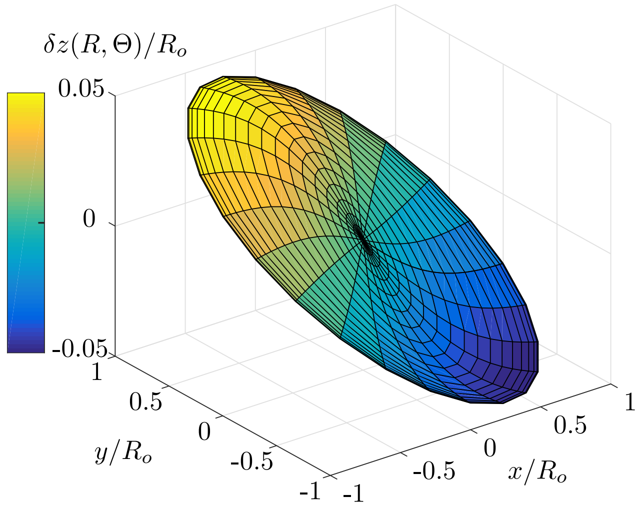


Figure 3: Visualisation of the incremental displacement of a cross section of a cylinder with a non-radial perturbation of a radially symmetric distribution of screw dislocations.

References:

[1] Sadik, S., & Yavari, A. (2016). Small-on-large geometric anelasticity. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 472*(2195), 20160659.

Swelling theory for poly-domain materials

L. Teresi

Università degli Studi Roma Tre, Italia

Abstract: We present a theoretical framework to model the complex chemo-mechanical response of poly-domain, swellable materials, accounting for their heterogeneous composition. Biological materials, which are often soft and hydrated, exemplify this class of materials.

These natural materials are synthesized/assembled in presence of solvent, and the biopolymers might be always stretched in the free-swollen state [1]; moreover, even at steady states, the solvent content and the network stretching might vary from a region to another.

Starting with the hypotheses underlying the well known Flory-Rehner model for mechano-diffusion [2], we have developed an augmented chemo-mechanical model that exploit the theory of nonlinear elasticity with large distortions [3], and is capable of describing the effects of swelling for non homogeneous materials; the same approach has been used to model self-contractile gels [4].

The theory was fully developed and tested on prototypical phenomena observed in edible matter, particularly in fruits and vegetables with soft cores and stiff skins.

Our results show that the onset of instabilities and wrinkle formation depends on various factors, including the overall geometry of the specimen and

the specific geometry of each domain. Also dynamics is fundamental, as dehydration or swelling might happen very slowly or very fast with respect to the characteristic time of the specimen considered.

Joint work with M. Curatolo and R.G.M. van der Sman.

References:

[1] R.G.M. van der Sman, Biopolymer gel swelling analyzed with scaling laws and Flory–Rehner theory, Food Hydrocolloids (2015).

[2] P. Flory, J. Rehner, Statistical mechanics of cross-linked polymer networks II. Swelling, J. Chem. Phys. (1943).

[3] M. Curatolo, S. Gabriele, L. Teresi, Swelling and growth: a constitutive framework for active solids, Meccanica (2017).

[4] A. Bernheim-Groswasser, G. Livne, P. Nardinocchi, F. Recrosi, L. Teresi, Interplay between activity, elasticity, and liquid transport in self-contractile biopolymer gels, Phys. Rev. E (2024).

An Eulerian formulation of a constrained variable thickness growing Cosserat shell

G. Tomassetti

Università degli Studi Roma Tre, Italia

Abstract: We propose an Eulerian formulation of constitutive equations for a growing shell. The director is constrained to remain normal to that surface, but it is allowed to have a variable length to model non-uniform thickness changes of the shell. Elastic measure of dilatation, distortional deformation, mean and Gaussian curvatures are determined by evolution equations that are independent of a reference or intermediate configuration. These evolution equations model homeostasis, which is the process of growth causing a tendency for these variables to approach their homeostatic values.

Joint work with M.B. Rubin.

Buckling and post-buckling of fiber-reinforced tissues

Yang Liu

University of Oxford, UK

Abstract: Many biological tissues are fiber-reinforced, enhancing their physical properties and exhibiting inherent anisotropy. The interplay of fiber stiffness, fiber orientation, and the elastic properties of the matrix on pattern formation and evolution in layered tissues still requires a comprehensive understanding. Here, we consider a paradigmatic model with a film coated to a half-space where either the film or the substrate is anisotropic. We explore both buckling and post-buckling behaviors within the framework of nonlinear elasticity. In particular, the Holzapfel-Gasser-Ogden model is used to capture anisotropy. An exact bifurcation for surface wrinkling is derived, from which it is found that fiber-reinforcement in the film can either suppress or promote surface instability, depending on the fiber orientation. By contrast, if the substrate is fiber-reinforced instead, surface wrinkles are always delayed, and a phase diagram for various instability modes is presented. To investigate post-buckling evolution, a weakly nonlinear analysis is performed to derive an amplitude equation applicable to both scenarios. A detailed parametric study unravels the effects of fiber orientation, fiber stiffness, and modulus ratio on the post-buckling solution. When the film is fiber-reinforced, it turns out that the bifurcation will always be supercritical if $r < 0.571$ where r is the ratio of the shear modulus of the substrate to that of the film. we identify an additional condition under which subcritical bifurcations can emerge. If the substrate is fiber-reinforced, the integrated impact of material anisotropy becomes more complicated. We further define an effective modulus ratio that incorporates the shear moduli of both layers and the anisotropy parameters, and illustrate a phase diagram for subcritical and supercritical bifurcations. Finally, we identify from our post-buckling solution where the fibers first experience compression and also plot the bifurcation diagram from which the evolution of the wrinkled amplitude is depicted. These analytical post-buckling solutions offer valuable insights into the morphological development biological tissues.

Some experimental, empirical, and modelling results in the bursty deformation of materials

G. Zanzotto
Università degli Studi di Padova, Italia

Abstract: We describe some recent experimental and empirical studies of the bursty strain behavior in the mechanics of crystalline materials,including their (ir)reversible structural transformations and their plastic deformation. We highlight some features of the underlying strain-avalanching, and present a related crystallographically-informed modeling, with the ensuing strain-intermittency results in different mechanical or thermal loading scenarios.

Smooth transition to helicoids in twisted cords

G. Zurlo
University of Galway, Ireland

Abstract: When a straight rubber cord is twisted while it is axially pre-stretched, it becomes unstable at a critical torque value, leading to the formation of intricate instability patterns. If the cord is sufficiently slender and the applied torque is increased gradually, a detailed analysis of the instability sequence reveals that initially, helical configurations emerge along its length. As the torque continues to increase, these helical shapes also destabilize, ultimately resulting in the formation of knots.

Investigating this sequence of instabilities through analytical methods is highly challenging due to geometric and constitutive nonlinearities. This work introduces an energy-based approach to studying the first stage of this instability process, the formation of an helicoid out of the straight, twisted state. Firstly, a dimensional reduction procedure allows to derive an approximate energy expression for a homogeneous helicoid in a slender, incompressible and

hyperelastic cord. This expression of the energy allows to explore simultaneously both straight-axis configurations and helical configurations under axial pre-stretch and torsion, and shows that the transition from a straight to a helical state occurs at a precise value of the critical torsional load. At the onset of instability, helicoidal configurations become energetically favorable, and they kick in smoothly with an amplitude that increases gradually as the applied torque is increased above the critical value.

This study transparently illustrates the energetic preference for helicoidal states in slender, incompressible cylinders, aligning with experimental observations on twisted rubber cords. The work is dedicated to Professor Giuseppe Saccomandi, a friend and mentor, whose figure exemplifies how Science can be pursued at the highest level with joyful enthusiasm and passion. This is joint work with Michel Destrade.