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The principle of maximum judo efficacy as a method of comparing two levers Flexional (Ude Hishigi Juji Gatame) and torsional-flexional (Ude Garami)

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Abstract

This project is an analysis of a judo arm bar.

Judo is an olympic sport that builds is foundation on having the best result with minimum effort.

The aim is to understand which angle is more effective to create an elbow or a shoulder dislocation executing an arm bar called Ude Garami (Entangled Arm Lock).

To demonstrate this aim we thought of an arm as two different beams using the equations of De Saint Venant to define the torque stress and the bending stress and Tresca's criterium in order to define an equivalent stress. We use the hypothesis of very slender bones in order to minimize the shear stress on elbow and shoulder.

It made us understand that Ude Garami *technique is similar to the one known as* Ude Hishigi Juji Gatame (Cross Arm Lock) *if the angle is equal to* π*. Also we understood which lever is the best to execute if the arm and the forearm have different moment of inertia.*

In conclusion, a summary graph describes all the calculations.

1. Introduction

Created by sensei Jigoro Kano in 1884 judo is a martial art, which origins reside in Jiu-Jitsu. The main task for a judoka is to throw the opponent to the ground using takedowns or making him/her tap one of his/her hands to the tatami through submission movements. The referring names to both players in a judo fight are *tori*, the one who executes the technique and *uke*, the one who suffers the technique.

Balance is the main concept of judo and links it to an engineering point of view: if you lose your balance, you lose the match.

Submission movements are the main topic of this paper and the focus will be on a very effective and complex lever, *Ude Garami*.

As we know in physics exist three different types of lever that are classified depending on where the fulcrum is located.

- Class 1: Fulcrum in the middle;
- Class 2: Resistance in the middle;
- Class 3: Effort in the middle.

2. Anatomy

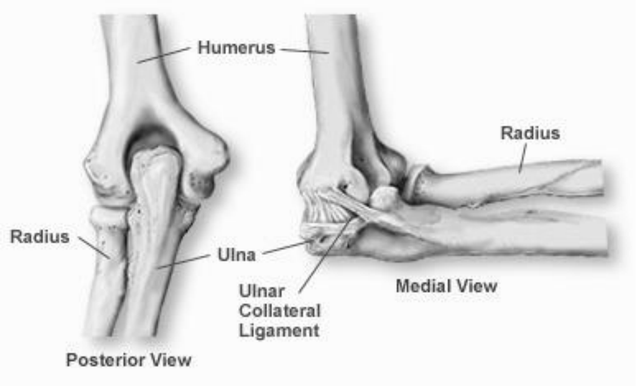
The arm bars are the levers specifically used in a judo competition, but to understand the lever we need to explain the anatomy of an upper limb.

Two different parts form an upper limb: the arm and the forearm.

The arm has a single bone called *humerus* whereas the forearm has two different bones, *ulna* and *radius*.

The distal part of the *humerus* and the proximal parts of *ulna* and *radius* form a hinge joint also known as elbow joint.

- The *capitulum* of the *humerus* articulates with the upper aspect of the head of the *radius;*
- The *throclea* of the *humerus* articulates with the *throclear notch* of the *ulna;*
- These two parts of the elbow joint are continuous with each other and share a common cavity with the proximal radio-ulnar joint.



Pic 1: The elbow joint

3 Ude Garami (Entangled Arm lock)

In *Ude Garami* technique *tori* grabs the wrist of *uke’*s already extended arm. Then, with his/her free hand, *tori* runs his/her other arm beneath the elbow of *uke*, the grasped-wrist arm, and clasps his/her other hand, the one that is holding u*ke* wrist, as shown in Pic.2.

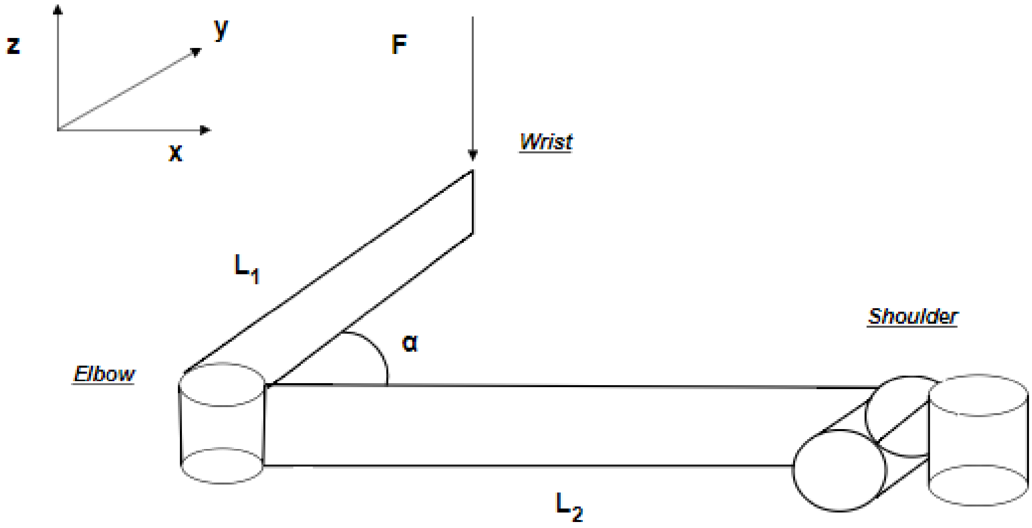
Pic. 2: Execution of Ude Garami

It was necessary to figure out how to describe this technique with a scheme.

In this particular case, starting from the head of the *humerus* we can assume it as a horizontal cylindrical hinge and a vertical cylindrical hinge. The elbow is a vertical cylindrical hinge and the wrist is not considered as a constraint.

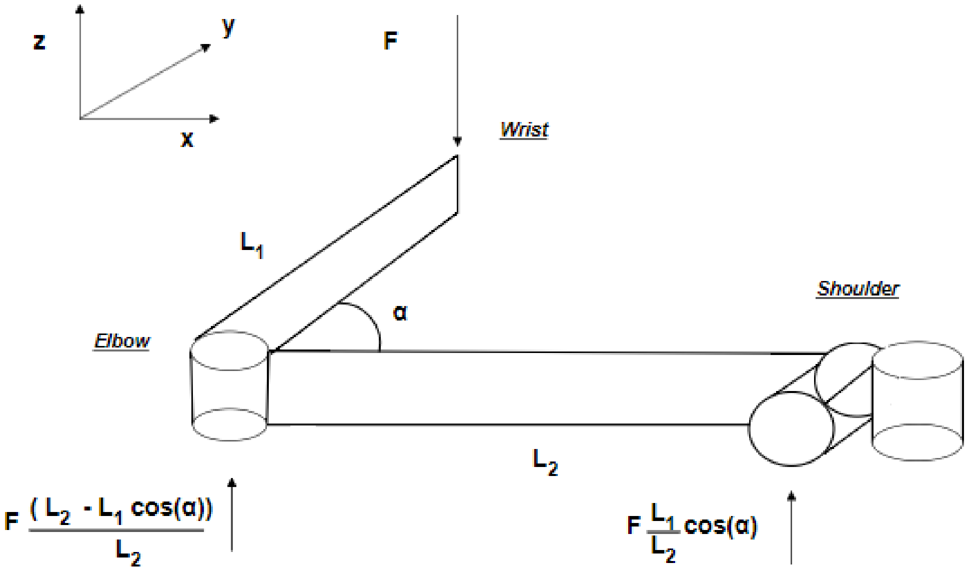
Last thing to define is a right-handed coordinate reference system.

In Pic.3, we can see our starting position.



Pic. 3: Ude Garami scheme

Some reaction forces will generate as shown in Pic.4.



Pic.4: Reaction Forces

3.1 Momentum

The aim of this paper is to understand how this particular arm bar works.

To do so we need to calculate all of the momentum that can create an elbow or a shoulder dislocation and then moving to the calculations of torque and bending stress.

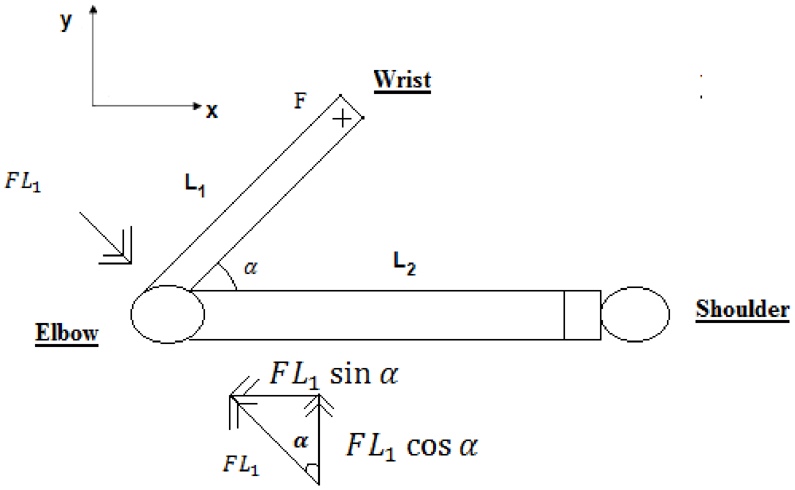
It is necessary to divide the elbow in a “right before” and a “right after” the joint.

Here some symbols that we will be used from now on:

Symbols	Meaning
$"-"$	right before the elbow joint
$"+"$	right after the elbow joint
$\boldsymbol{L_1}$	Length of the Forearm
$\boldsymbol{L_2}$	Length of the Arm
$\boldsymbol{F}$	Force
$\boldsymbol{\alpha}$	Angle
$\boldsymbol{A}$	Area
$\boldsymbol{I}$	Moment of Inertia
$\boldsymbol{I_p}$	Polar moment of Inertia
$\boldsymbol{M_b^e}$	Bending momentum (elbow)
$\boldsymbol{M_t^e}$	Torque momentum (elbow)
$\boldsymbol{M_b^s}$	Bending momentum (shoulder)
$\boldsymbol{M_t^s}$	Torque momentum (shoulder)
$\boldsymbol{\sigma_b^e}$	Bending stress (elbow)
$\boldsymbol{\tau_t^e}$	Torque stress (elbow)
$\boldsymbol{\tau_t^s}$	Torque stress (shoulder)
$\boldsymbol{\tau_T^e}$	Shear Stress (elbow)
$\boldsymbol{\tau_T^s}$	Shear Stress (shoulder)
$\boldsymbol{\sigma_{eq}^e}$	Equivalent Stress (Tresca’s law)
$\boldsymbol{\sigma_{eq}^s}$	Equivalent Stress (Tresca’s law)

Table 1: Symbols

A sight from above:



Pic.5: Momentum on the "XY" plane

$M_b^{e-} = FL_1$ $\quad$ $M_t^{e+} = FL_1 \sin\alpha$

$M_b^{e+} = -FL_1 \cos\alpha$

$M_b^s = -FL_1 \cos\alpha + F\frac{L_1}{L_2}\cos\alpha\, z$ if $z = L_2$, $M_b^s = 0$

The shoulder does not receive any bending momentum.

$M_t^s = -FL_1 \sin\alpha$

About bending and torque stress:

$$\sigma_b^{e-} = \frac{M_b^{e-}}{I_x} r = \frac{FL_1}{I_1} r_1$$

$$\sigma_b^{e+} = \frac{M_b^{e+}}{I_x} r = -\frac{FL_1 \cos\alpha}{I_1} r_1$$

$$\tau_t^{e+} = \frac{M_t^{e+}}{I_p} r = \frac{FL_1 \sin\alpha}{2I_2} r_2$$

$$\tau_t^{s} = \frac{M_t^{s}}{I_p} r = -\frac{FL_1 \sin\alpha}{2I_2} r_2$$

$$\sigma_{eq}^{e+} = \sqrt{(\sigma_b^{e+})^2 + 4(\tau_t^{e+} + \tau_T^{e+})^2} = \sqrt{(-\frac{FL_1 \cos\alpha}{I_1} r_1)^2 + 4(\frac{FL_1 \sin\alpha}{2I_2} r_2 + \frac{4}{3}\frac{FL_1 \cos\alpha}{L_2 A})^2}$$

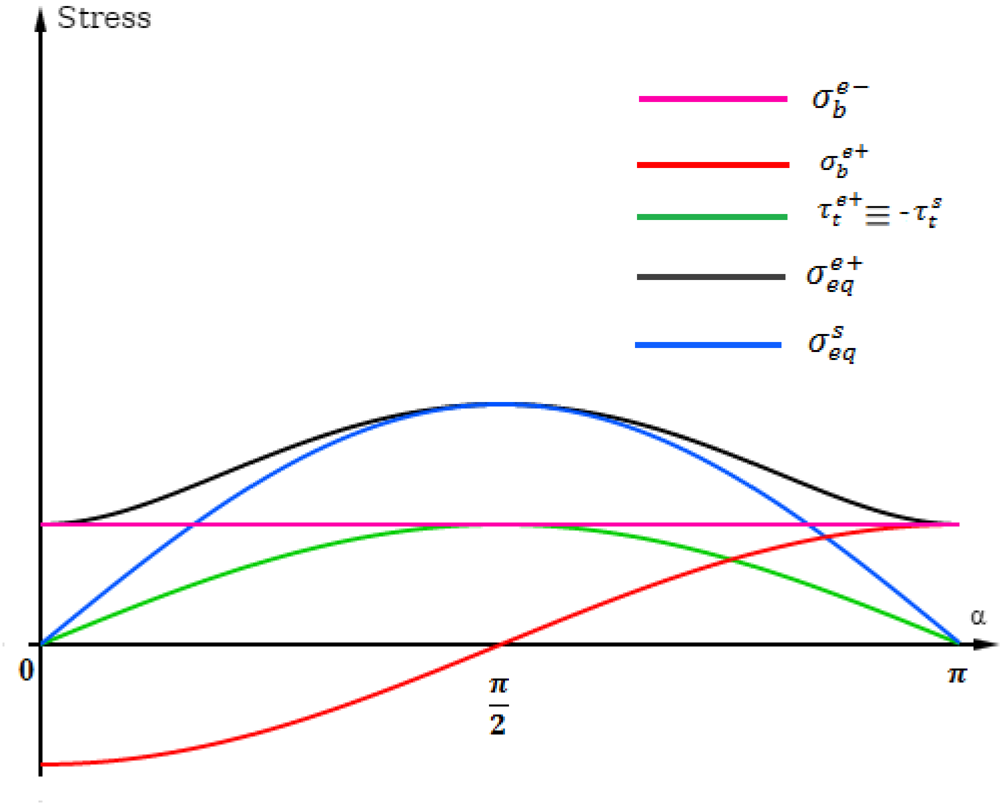
$$\sigma_{eq}^{s} = \sqrt{4(\tau_t^{s} + \tau_T^{s})^2} = \sqrt{4\left(-\frac{FL_1 \sin\alpha}{2I_2} r_2 + \frac{4}{3}\frac{T}{A}\right)^2} = \sqrt{4\left(-\frac{FL_1 \sin\alpha}{2I_2} r_2 + \frac{4}{3}\frac{FL_1 \cos\alpha}{L_2 A}\right)^2}$$

3.2 Cases

What is the meaning of angle α?

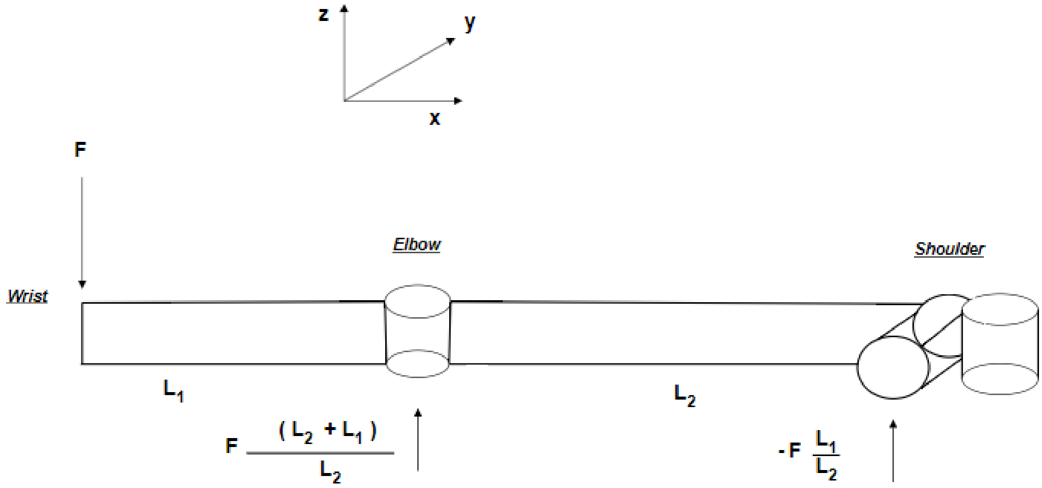
The angle defines torque and bending stress as shown in Pic.6.

The bending stress on e^- is a constant. On e^+ the bending stress starts with a negative value if the angle is equal to 0. The torque stress depends on the $\sin\alpha$ as the σ_{eq}^{s}. The equivalent stress on the elbow (σ_{eq}^{e+}) is a combination between bending and torque stress.



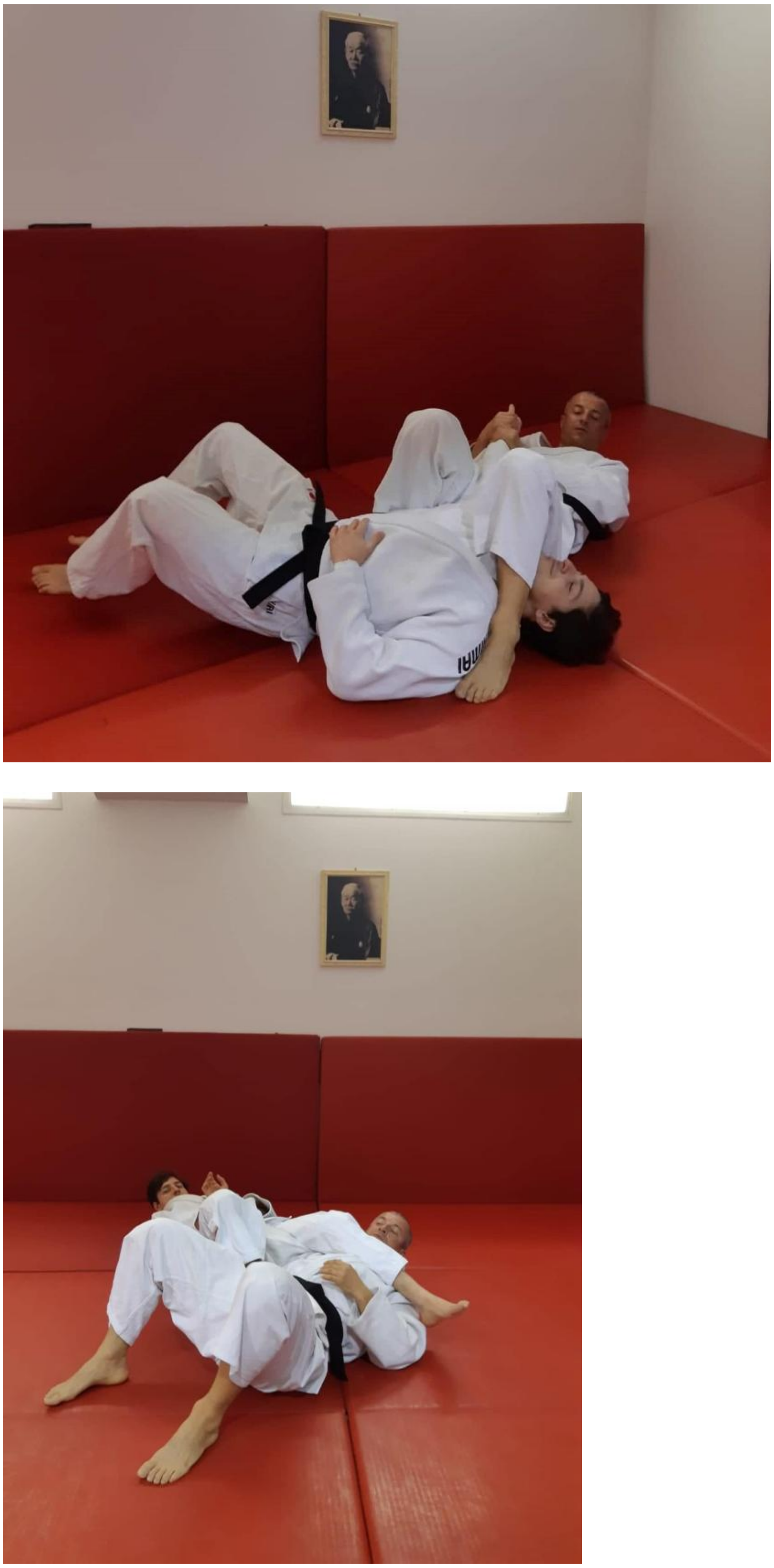
Pic.6: Bending Stress, Torque Stress, Equivalent stress on elbow and shoulder.

- **Case one: $\alpha = \pi$ or $\alpha = 0$**



Pic.7: Case one: $\alpha = \pi$

This $\sigma_{eq}^{s} = \sqrt{4(-\frac{4}{3}\frac{FL_1}{L_2A})^2}$, $\sigma_{eq}^{e+} = \sqrt{(\frac{FL_1}{I_1}r_1)^2 + 4(-\frac{4}{3}\frac{FL_1}{L_2A})^2}$, means that *tori* is not twisting the shoulder of *uke* so we are in front of another type of arm lock, called *Ude Hishigi Juji Gatame (Cross Arm Lock).*



Pic.8: Execution of Ude Hishigi Juji Gatame

The elbow is in pure bending and there is no torque.

- **Case two: $\boldsymbol{\alpha = \frac{\pi}{2}}$**

Opposite to the first case there are these calculations,

$\sigma_{eq}^{s} = \frac{FL_1}{I_2} r_2$ and $\sigma_{eq}^{e+} = \frac{FL_1}{I_2} r_2$.

$\tau_t^{e+} = \frac{FL_1}{2I_2} r_2$ and $\tau_t^{s} = -\frac{FL_1}{2I_2} r_2$, so the elbow and the shoulder are in pure torque. The twisting momentum will create a dislocation.

3.3 What if $I_1 \neq I_2$?

Introducing the slenderness: $\lambda^2 = \frac{L^2 A}{I}$

From now on, we assume that bones are very slender. With this hypothesis, we can elide the shear factor on the elbow and on the shoulder.

Assuming $\frac{r_2}{I_2} = \beta \frac{r_1}{I_1}$

$$\sigma_{eq}^{e+} = \sqrt{\frac{F^2 L_1^2 r_1^2}{I_1^2} \left(cos^2(\alpha) + \beta^2 sin^2(\alpha)\right)}$$

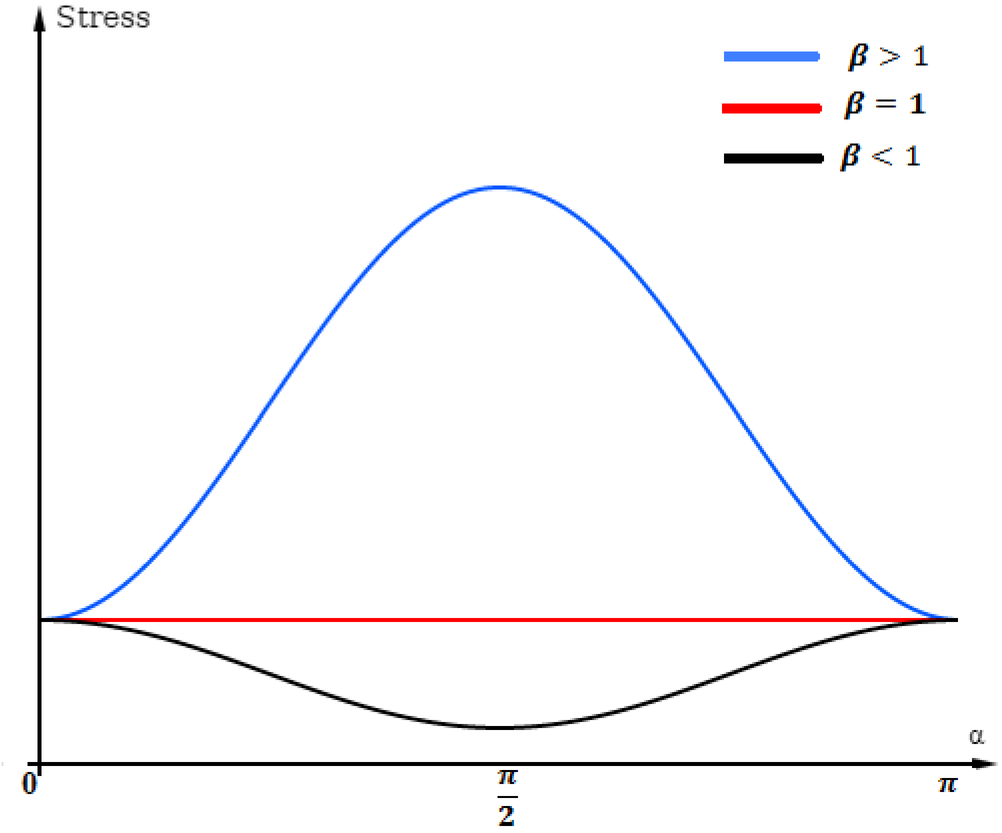
$$f = \frac{(\sigma_{eq}^{e+})^2}{\frac{F^2 L_1^2 r_1^2}{I_1^2}} = cos^2(\alpha) + \beta^2 sin^2(\alpha)$$

This equation depends on the angle so let us try to understand if we have some maximum or minimum.

$$\frac{df}{d\alpha} = 2\cos\alpha\,(-\sin\alpha) + \beta^2 2\sin\alpha\cos\alpha = (\beta^2 - 1)2\sin\alpha\cos\alpha = (\beta^2 - 1)\sin 2\alpha$$

Even though it is much more important to understand the meaning of β.

β describes how much bigger is the arm compared to the forearm, so the need is to divide it in three different cases:



Pic.9: Differences in moment of inertia due to $\boldsymbol{\beta}$ on $\boldsymbol{e^+}$.

- $\boldsymbol{\beta} > 1$

  We will have a minimum for $\alpha = \pi \; and \; \alpha = 0$ and a maximum for $\alpha = \frac{\pi}{2}$, with a periodicity of π. It means that if the arm is bigger than the forearm, *tori* will need to execute *Ude Garami* with an angle of $\frac{\pi}{2}$ to maximize the effect on the elbow;

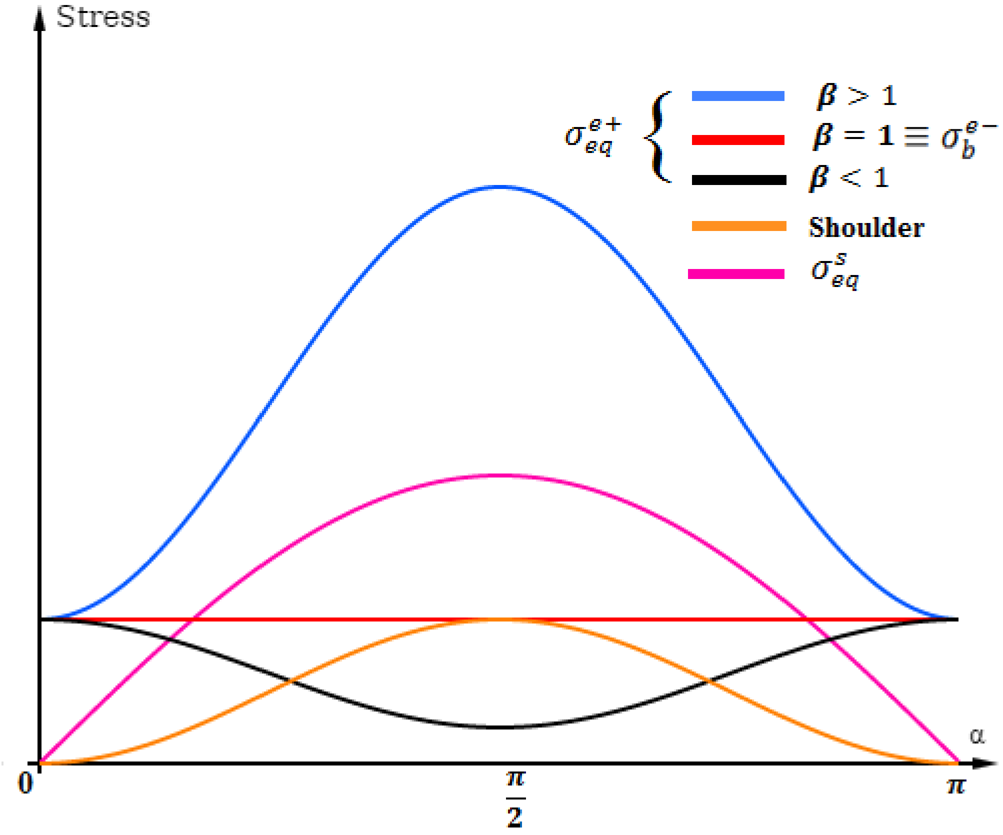
- $\boldsymbol{\beta = 1}$

  In this particular case, the forearm is the same size of the arm, so the effect will be the same in all three angles;

- $\boldsymbol{\beta} < 1$

  When a forearm is bigger than the arm, to maximize the effect, it is better for *tori* to execute an *Ude Hishigi Juji Gatame* in order to dislocate the elbow.
  If the angle is $\frac{\pi}{2}$, *Ude Garami* will have minimum effect due to the resistance of the forearm.

4 Conclusion



Pic.10: Summary graphic with equivalent stress on elbow and shoulder.

- If $\boldsymbol{\beta} > 1$ the arm will suffer a dislocation of both shoulder and elbow, *tori* needs to execute *Ude Garami* with an angle of $\frac{\pi}{2}$ in order to maximize the lever;
- If $\boldsymbol{\beta} = \mathbf{1}$ the elbow will suffer the same stress in every angle but in order to create a torque stress to the shoulder, still, *tori* needs to execute it at an angle of $\frac{\pi}{2}$;
- If $\boldsymbol{\beta} < 1$ the elbow will have a maximum at π, it means that *uke*'s forearm is bigger than his/her arm, so *Ude Hishiji Juji Gatame* is more effective than *Ude Garami*.

Both of the lever are very technical and complicated, it is important for Uke to tap at the start of the pain in order to prevent some serious injuries. This work helped us understand the idea behind Judo, with just an application of force on the wrist we can create movements in all the different sections of the arm even if the opponent has bigger forearms instead of bigger arms.

Pic. 11: Prof. Pugno and Giuseppe

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