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Mechanical modelling of viscoelastic hierarchical suture joints and their optimization and auxeticity

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Abstract

It has been widely recognised that suture joints play a fundamental role in the exceptional mechanical properties of many biological structures like the cranium and ammonite fossil shells. Up to date, many efforts have been devoted to investigate suture interfaces to predict the relations between the characteristics of its two phases, the teeth and the interface layer, and the structure effective properties. However, in very few works the viscoelasticity of the suture components has been taken into account. To provide a contribution in this limitedly explored research area, this paper describes the mathematical formulation and modelling technique leading to explicit expressions for the effective properties of viscoelastic suture joints with a general trapezoidal waveform. It emerges a strong influence of the suture geometric and mechanical characteristics on the effective properties and a parametric analysis reveals that an auxetic behavior can be obtained simply by tailoring the suture parameters. The effects of adding hierarchy into the above system are also explored and closed-form relations for the effective moduli are derived. Optimal levels of hierarchy can be identified and, similarly to the non-hierarchical case, an auxetic behavior emerges for particular values of the suture parameters. Finally, an extension of the theory to the sinusoidal sutures in biology is reported. By considering the examples of the cranium and the woodpecker beak, it emerges that the first is optimized to obtain high stiffness, while the second to obtain high energy dissipation levels.

Keywords: viscoelasticity, hierarchy, suture joints, effective properties, loss factor, auxeticity

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1. Introduction

Suture joints with different geometries are commonly found in biology from the micro- to the macro-length scales. Examples include the carapace of the turtle [1, 2], the woodpecker beak [3], the cranium [4], the seedcoat of the Portulaca oleracea [5] and Panicum miliaceum [6] and ammonite fossil shells [7], among others. The suture joint architecture, where two interdigitating stiff components (the teeth) are joined by a thin compliant seam (the interface layer), allows an high level of flexibility and is the key factor for the accomplishment of biological vital functions such as respiration, growth, locomotion and predatory protection [8-10]. From a mechanical point of view, it has computationally and/or experimentally been demonstrated that this particular configuration endows an excellent balance of stiffness, strength, toughness, energy dissipation and a more efficient way to bear and transmit loads [11, 12, 14, 15, 32]. Emblematic is the case of the leatherback sea turtle, a unique specie of sea turtle having the capacity to dive to a depth of 1200 m [20]. As reported in [20], this is due to the particular design of the turtle carapace, where an assemblage of bony plates interconnected with collagen fibers in a suture-like arrangement is covered by a soft and stretchable skin. The combination of this two elements provides a significant amount of flexibility under high hydrostatic pressure, as well as exceptional mechanical functionality in terms of stiffness, strength and toughness, being the collagenous interfaces an efficient crack arrester. These fascinating performances, however, are strongly affected by the suture geometry that, in nature, varies among and within species as a result of an evolutionary process [16-18]. This is confirmed in [9, 19], where it emerges that an increased level of interdigitation found in the cranial sutures of different mammalian species leads to an increase in the suture bending strength and energy storage [9, 19]. Similarly, [24] explains not only how the high sinuosity and complexity of the suture lines in ammonites are the result of an evolutionary response to the hydrostatic pressure, but also that the stresses, displacements and deformations significantly decrease with the level of complexity. Analogous results are obtained in [25], which seeks to clarify the functional significance of the complex suture pattern in ammonites by means of a fractal analysis-based technique. The geometric characterisation of different types of biological suture joints is also reported in [21-23].

Starting from the pioneering contributions in [21–25], many authors studied the mechanics of suture joints and a vast literature has flourished in recent years. Simple relations between the geometric parameters (tooth shape, tooth tip angle, volume fraction of the teeth) and effective

mechanical properties (stiffness, strength, fracture toughness) of trapezoidal suture joints are reported in [11, 26], together with a discussion on the role of geometry on the suture mechanical behavior. Experimental results are illustrated in [27–30] for the characterisation of the deformation and failure mechanism of 3D-printed suture joints with different geometries. An enhancement in the overall toughness and load-bearing capacity have been observed in the case of a sinusoidal waveform, suggesting a possible way to improve suture joints where the major structure is made of a brittle polymer [28]. An application of the theory in [11, 26] to the wavy-patterned sutures of the common millet (*Panicum miliaceum*) seedcoat is discussed in [31, 32], where the important role played by suture interfaces in resisting indentation loads is revealed [31]. Inspired by the hierarchical suture joints found in ammonite fossil shells, the authors of [7] propose a two-dimensional deterministic fractal model to investigate the role of hierarchy on the stiffness, strength and failure mechanism of triangular sutures. The hierarchical configuration, in particular, is obtained by superimposing a self-similar waveform with a shorter wavelength on each wave of the former profile. An increasingly complex suture pattern is thus created and, in line with [11, 26], an energetic approach leads to analytical relation between its effective properties and its geometric and mechanical features. More recently, a multiscale fracture analysis of staggered composites consisting of wave tablets is proposed in [33], where the influence of the tablets waveform on the effective fracture resistance and damage tolerance is clarified. Finally, the analysis of the damping performances of viscoelastic composites with sutural interfaces is presented in [12], where closed-form expressions for the loss factor, loss modulus and storage modulus are reported for the case of triangular and trapezoidal sutures.

As clearly emerges, in the literature many works are devoted to the mechanical characterisation of suture joints, with a special attention on the role of the suture geometry on the overall behavior. Surprisingly, in spite of the proven benefits distinguishing the biological hierarchical sutures, very few investigations are currently available on this topic. In addition, very few investigations concern suture joints where one or both phases display a viscoelastic material response. To make a contribution to this incomplete research area and to provide some useful tools for practical applications, this paper focuses on the effects of adding hierarchy into a two-dimensional viscoelastic suture joint having a general trapezoidal configuration. The work is organised in 8 sections, including this introduction. Initially, based on the elastic-viscoelastic correspondence principle, Section 2 illus-

trates the mathematical formulation and modelling technique leading to closed-form relations for the effective moduli (Young's moduli, shear modulus and Poisson's ratios) and damping properties (loss factor, storage moduli and loss moduli). Considerations about the influence of the suture geometric and mechanical characteristics on the effective properties are presented in Section 3. In Section 4, the theory is extended to hierarchical sutures and explicit expressions for the effective viscoelastic properties are derived. A parametric analysis to investigate how the effective behavior is affected by the suture features in the hierarchical case is summarised in Sections 5 and 6. Optimal levels of hierarchy can be identified, together with the possibility to achieve auxeticity for particular suture configurations. As a real-case example, Section 7 illustrates the extension of the model to evaluate the effective stiffness and damping properties of the sinusoidal patterned sutures in biology. Two cases are compared: the cranium and the woodpecker beak. The first is found to be optimized for obtaining high stiffness, the second for obtaining high energy dissipation levels. As a conclusion, Section 8 summarises the main findings. To the authors best knowledge, this study is the first to explore a suture joint by incorporating both the two fundamental ingredients of hierarchy and viscoelasticity.

2. Viscoelastic suture joints: theoretical description and effective properties

2.1. Problem statement: basic concepts and assumptions

From a mechanical point of view, suture joints can be represented as a composite material including two interdigitating stiff phases (the teeth) joined by a thin compliant element (the interface layer) along the seam line (Fig. 1). Due to physiological reasons as respiration, growth or motion of the biological system [11], suture joints are often subjected to a in-plane far-field external tension $\hat{\sigma} = (\hat{\sigma}_{11}, \hat{\sigma}_{22}, \hat{\sigma}_{12})$ acting along the far-field boundaries $\partial\Omega$ (Fig. 1a). These loads are transmitted across the interface through a combination of interfacial shear and normal stresses given, respectively, by

$$\begin{cases} \tau_I = \tau_I \left(\hat{\sigma}, \, \alpha_S, \, \beta_S, \, \vartheta_S \right) \\ \sigma_I = \sigma_I \left(\hat{\sigma}, \, \alpha_S, \, \beta_S, \, \vartheta_S \right), \end{cases}$$
(1)

with $\alpha_S, \beta_S, \vartheta_S$ the geometric parameters defining the interface profile (Figs. 1c, 2).



Figure 1: (a) Schematic representation of a general trapezoidal suture joint indicating (b) the stresses acting at the interface and (c) the most relevant geometric parameters.



If we impose the equilibrium conditions at the interface, it emerges that the tangential and normal components of stress within the two phases satisfy the following relation (Fig. 1b):

$$\begin{cases} \tau_T = \tau_L = \tau_I \\ \sigma_T = \sigma_L = \sigma_I, \end{cases}$$
(2)

being $(\cdot)_T$ and $(\cdot)_L$, on order, the stresses within the teeth and the interface layer. In line with [11], the effective mechanical properties of suture joints can be obtained by focusing on the interdigitating area (Fig. 1c), whose boundaries are now coinciding with $\partial\Omega$, where the tension $\hat{\sigma}$ is applied. Also, due to the periodicity of the configuration, the analysis can be restricted to the Representative Volume Element (RVE), as highlighted in Figure 1c. Its geometry is described by two independent non-dimensional geometric parameters: the volume fraction of the teeth, ϕ_T , and the tooth tip angle, β_S , given by [11]:

$$\phi_T = 1 - \frac{2h_L}{\lambda_S}, \qquad \tan \beta_S = \frac{\phi_T \lambda_S}{2A_S} = \frac{\lambda_S - 2h_L}{2A_S}, \qquad (3)$$

with A_S and λ_S , respectively, the amplitude and wavelength of the suture, h_L the thickness of the interface layer measured in the e_1 direction (Fig. 1c).

Regarding the constituents, i.e., the teeth and the interface layer, two simplifying hypothesis are made. The first is to consider the teeth and the interface layer perfectly bonded at the slant interfaces, the second is to assume the flat tip region unbonded. As discussed in [11], these assumptions do not affect the validity of the analysis. Both the teeth and the interface layer are also assumed homogeneous and their mechanical response is linear elastic, in the case of the teeth, and viscoelastic, in the case of the interface layer. The latter, in particular, is described by the Kelvin-Voigt model [12]:

$$\tau_L = G_L \gamma + \eta \dot{\gamma} \quad \text{shear behavior}$$

$$\sigma_L = E_L \varepsilon + \xi \dot{\varepsilon} \quad \text{normal behavior},$$
(4)

with G_L and E_L , respectively, the layer shear modulus and Young's modulus, η the viscosity coefficient for shear deformation γ , ξ the viscosity coefficient for normal deformation ε and $(\dot{\cdot}) := \frac{d(\cdot)}{dt}$.



Figure 2: Suture geometric profiles corresponding to different values of the angles α_S , β_S , ϑ_S : (a) trapezoidal, (b) rectangular, (c) triangular, (d) anti-trapezoidal.



2.2. Effective properties

Closed-form relations for the effective in-plane Young's moduli, shear modulus and Poisson's ratios accounting the viscoelastic effect can be derived in the frequency domain by alternately prescribing a periodic loading condition

$$\hat{\sigma}_{1} = \begin{bmatrix} \hat{\sigma}_{11} \neq 0 \\ \hat{\sigma}_{22} = 0 \\ \hat{\sigma}_{12} = 0 \end{bmatrix}, \quad \hat{\sigma}_{2} = \begin{bmatrix} \hat{\sigma}_{11} = 0 \\ \hat{\sigma}_{22} \neq 0 \\ \hat{\sigma}_{12} = 0 \end{bmatrix}, \quad \hat{\sigma}_{3} = \begin{bmatrix} \hat{\sigma}_{11} = 0 \\ \hat{\sigma}_{22} = 0 \\ \hat{\sigma}_{12} \neq 0 \end{bmatrix}, \quad (5)$$

described by

$$\hat{\sigma}_{ij} = \Gamma_{ij} e^{i\omega t}, \qquad i, j = 1, 2, \tag{6}$$

with Γ_{ij} the amplitude and ω the circular frequency of a time-harmonic plane wave propagating along the medium.

When dealing with viscoelastic materials, it is convenient to use the elastic-viscoelastic correspondence principle [36], according to which the solution techniques developed for the linear elastic framework can be directly applied for viscoelastic media simply by replacing the elastic material parameters with their frequency-dependent complex counterparts. A classical mechanics-based analysis [11, 12] can be thus applied, providing, in the loading condition $\hat{\sigma}_1$, the complex effective Young's modulus in the e_1 direction

$$E_1 = \left(\frac{\lambda_S - 2h_L}{\lambda_S E_T} + \frac{2h_L}{\lambda_S} \left(\frac{\cos^2 \alpha_S \sin^2 \alpha_S}{\tilde{G}_L} + \frac{\cos^4 \alpha_S}{\tilde{E}_L}\right)\right)^{-1} \equiv E_1' + iE_1'' \tag{7}$$

and the related Poisson's ratio

$$\nu_{12} = -\frac{\lambda_S}{2A_S} \left(\frac{-\frac{\nu_T}{E_T} A_S - \frac{\tilde{\nu}_L}{\tilde{E}_L} \frac{h_L}{\tan \beta_S} + h_L \cos^3 \alpha_S \sin \alpha_S \left(\frac{1}{\tilde{G}_L} + \frac{1}{\tilde{E}_L}\right)}{\frac{\lambda_S}{2E_T} + h_L \cos^2 \alpha_S \left(\frac{\sin^2 \alpha_S}{\tilde{G}_L} + \frac{\cos^2 \alpha_S}{\tilde{E}_L}\right)} \right), \tag{8}$$

with E_T and ν_T , respectively, the Young's modulus and Poisson's ratio of the teeth, $\tilde{G}_L \equiv G_L + i\omega\eta$, $\tilde{E}_L \equiv E_L + i\omega\xi$ and $\tilde{\nu}_L$ the frequency-dependent shear modulus, Young's modulus and Poisson's ratio of the interface layer. According to the experimental observations in [38], the frequency dependency of $\tilde{\nu}_L$ can be neglected and it is thus possible to assume $\tilde{\nu}_L \approx \nu_L$, being ν_L the constant,

frequency-independent elastic Poisson's ratio of the interface layer. Similarly, the loading state $\hat{\sigma}_2$ yields the complex effective Young's modulus in the e_2 direction

$$E_2 = \frac{\lambda_S - 2h_L}{\lambda_S} \left(\frac{\Psi}{E_T} + \frac{h_L (\lambda_S - 2h_L)}{2A_S^2} \left(\frac{\cos^4 \alpha_S}{\tilde{G}_L} + \frac{\cos^2 \alpha_S \sin^2 \alpha_S}{\tilde{E}_L} \right) \right)^{-1} \equiv E_2' + i E_2'' \tag{9}$$

and the related Poisson's ratio

$$\nu_{21} = -\frac{\left(-\frac{\nu_T}{E_T}A_S - \frac{\tilde{\nu}_L}{\tilde{E}_L}\frac{h_L}{\tan\beta_S} + h_L\cos^3\alpha_S\sin\alpha_S\left(\frac{1}{\tilde{G}_L} + \frac{1}{\tilde{E}_L}\right)\right)\left(1 + \frac{2h_L}{\lambda_S}\left(\Phi_1 - 1\right)\right)}{\left(\frac{h_L}{2A_S}\frac{\Phi_1}{E_T} + \frac{A_S}{(\lambda_S - 2h_L)}\frac{\Psi}{E_T}\right)(\lambda_S + 2h_L\Phi_1)}, \quad (10)$$

with

$$\Psi := \begin{cases} 1 & \text{triangular} \\ 2/3 & \text{rectangular} \\ 2 - \frac{1}{\psi_{\alpha\beta}} + \frac{(1 - \psi_{\alpha\beta})^2}{2\psi_{\alpha\beta}^2} \ln\left(\frac{1 + \psi_{\alpha\beta}}{1 - \psi_{\alpha\beta}}\right) & \text{trapezoidal, anti-trapezoidal,} \\ \psi_{\alpha\beta} := \frac{\tan \alpha_S}{\tan \beta_S} \end{cases}$$
(12)

and

$$\Phi_1 := E_T \left(\frac{\cos^2 \alpha_S \sin^2 \alpha_S}{\tilde{G}_L} + \frac{\cos^4 \alpha_S}{\tilde{E}_L} \right).$$
(13)

Finally, the loading condition $\hat{\sigma}_3$ leads to the complex effective shear modulus

$$G_{12} = \frac{(\lambda_S - 2h_L) \Phi_2}{\left(\frac{\Phi_2}{4\Phi_3} \left(3 + \frac{5\Phi_3}{4E_T \tan^2\beta_S}\right) + \frac{\lambda_S \tan \alpha_S}{2A_S} \left(\frac{2h_L}{\lambda_S \psi_{\alpha\beta}} + \frac{3\tilde{G}_L}{2\Phi_3} \left(1 + \frac{3\Phi_3}{4E_T \tan^2\beta_S}\right)\right)\right) \lambda_S}$$
(14)
$$\equiv G'_{12} + iG''_{12},$$

where, to simplify the notation,

$$\Phi_2 := \tilde{G}_L \tan^2 \alpha_S + \tilde{E}_L, \qquad \Phi_3 := \frac{G_T \left(\lambda_S - 2h_L\right) + 2\tilde{G}_L h_L}{\lambda_S}, \qquad (15)$$

and G_T the shear modulus of the teeth.

In Equations (7), (9), (14), $(\cdot)'$ and $(\cdot)''$ stand, respectively, for the storage and loss moduli of the suture joint, whose ratio defines the loss tangent

$$\tan \delta_1 \equiv \frac{E_1''}{E_1'}, \quad \tan \delta_2 \equiv \frac{E_2''}{E_2'}, \quad \tan \delta_{12} \equiv \frac{G_{12}''}{G_{12}'}, \tag{16}$$

a useful parameter giving a measure of the ratio of the energy lost to the energy stored in a cyclic deformation [39]. This quantity allows us to evaluate the damping ability of the investigated system.

3. Discussion: the influence of the suture characteristics in the effective properties

According to the above formulation, this section aims at understanding how the suture geometric and mechanical parameters affect the effective properties. The analysis involves a suture joint having wavelength $\lambda_S = 15$ mm, amplitude $A_S = 15$ mm and where the teeth have Young's modulus $E_T = 6$ GPa, Poisson's ratio $\nu_T = 0.3$ and shear modulus $G_T = 2.3$ GPa. The interface layer has Young's modulus, $E_L = 10^{-2}E_T$, shear modulus $G_L = 10^{-2}G_T$, Poisson's ratio $\nu_L = 0.3$ and viscosity coefficients $\eta = \xi = 5 \text{ Pa} \cdot \text{s}$. Four different configurations are investigated (Figs. 3–5), corresponding to different values of the ratio α_S/β_S : -0.6, -0.3, 0, 1 that provide a suture joint having, respectively, an anti-trapezoidal, trapezoidal, rectangular and triangular profile (Fig. 2).

In general, for a fixed value of the teeth volume fraction, $\phi_T = 0.75$, Figures 3a, 3b, 3c suggest that increasing the tip angle β_S leads to an increase in the normalised Young's modulus E'_1/E_T , which is more significant for $\alpha_S/\beta_S = 1$, and to a decrease in the normalised Young's modulus E'_2/E_T . In terms of E'_2/E_T , the tooth tip angle has little influence for $\beta_S > 40^\circ$ while, for small values of β_S , E'_2/E_T increases very rapidly, especially in the case of the triangular profile (Fig. 3b).

A slightly different trend emerges in the case of the normalised shear modulus G'_{12}/E_T : an initial rapid increase for small values of β_S , followed by a peak region and a final decrease. However, as illustrated in Figure 3c, to the triangular ($\alpha_S/\beta_S = 1$) and trapezoidal ($\alpha_S/\beta_S = 0.3$) profiles correspond an higher peak value of G'_{12}/E_T than that which occurs to the rectangular ($\alpha_S/\beta_S = 0$) and anti-trapezoidal ($\alpha_S/\beta_S = -0.6$) ones. Figure 3c also suggests that changing the value of α_S/β_S provides a shear stiffness G'_{12}/E_T reaching the peak value at different β_S . For example, varying α_S/β_S from 1 to 0.3 results in G'_{12}/E_T exhibiting a peak value, respectively, at $\beta_S \approx 11^\circ$



Figure 3: The influence of the tooth tip angle β_S (first row) and teeth volume fraction ϕ_T (second row) in the normalised effective stiffness: (a), (d) Young's modulus E'_1/E_T ; (b), (e) Young's modulus E'_2/E_T ; (c), (f) shear modulus, G'_{12}/E_T .



and $\beta_S \approx 15^{\circ}$. Regarding the effective Poisson's ratios ν_{12} and ν_{21} , it emerges a 'Poisson's switch' from positive to negative values, obtained by varying the angle β_S (Figs. 4a, 4b). However, the value of β_S at which the 'Poisson's switch' occurs is affected by α_S/β_S . For make it more clear, let us focus on Figure 4a and let us consider the curves for $\alpha_S/\beta_S = -0.6$ and $\alpha_S/\beta_S = 0.3$ leading, respectively, to the anti-trapezoidal and trapezoidal profiles. In the first case, the 'Poisson's switch' occurs at $\beta_S = 20^{\circ}$ while, in the second, at $\beta_S \approx 47^{\circ}$. Similar considerations apply for ν_{21} (Fig. 4b), which experiences a 'switch' from positive to negative values at different β_S , depending on the suture geometry. These findings, in accordance with [5, 11], reveal the possibility to obtain a large control in the mechanics of suture joints and, in particular, an auxetic behavior can be obtained simply by tailoring the geometric characteristics of the two constituents.



Figure 4: The influence of the tooth tip angle β_S (first row) and teeth volume fraction ϕ_T (second row) in the effective Poisson's ratios: (a), (c) ν_{12} ; (b), (d) ν_{21} .

Analogous considerations can be derived by focusing on the loss tangent $\tan \delta_1$, $\tan \delta_2$ and $\tan \delta_{12}$ illustrated in Figures 5a, 5b, 5c. Independently of the ratio α_S/β_S , the loss tangent appears to be strongly affected by the tooth tip angle β_S and, even a small increase of β_S , leads to an high increase in $\tan \delta_1$, $\tan \delta_2$ and $\tan \delta_{12}$.



Figure 5: The influence of the tooth tip angle β_S (first row) and teeth volume fraction ϕ_T (second row) in the loss tangent: (a), (d) $\tan \delta_1$; (b), (e) $\tan \delta_2$; (c), (f) $\tan \delta_{12}$.



Finally, regarding the influence of the teeth volume fraction ϕ_T , Figures 3d, 3e, 3f reveal that, for a fixed value of the tooth tip angle $\beta_S = 30^\circ$, increasing ϕ_T leads to an increase of E'_1/E_T , E'_2/E_T and G'_{12}/E_T , regardless the geometric features of the suture profile. A common feature of Figures 5d, 5e, 5f is the higher dissipation achieved for small values of ϕ_T . This result can be explained by the fact that the lower ϕ_T , the higher will be the percentage of the viscoelastic interface layer, which is the element that dissipates energy rate.

4. Hierarchical extension

4.1. Overview

Let us imagine to modify the viscoelastic suture joint analysed in Section 2 by replacing the homogeneous interface layer with a structural element having the same periodic configuration of the starting system, i.e., two interdigitating stiff phases joined by a compliant interface layer. This modification can be iterated at successively smaller length scales, creating ever-finer structural detail while preserving the structure overall geometry. The resulting system, which will be referred to as viscoelastic hierarchical suture joint, is illustrated in Figure 6, representing the general case of a level-[n] hierarchical suture joint, where n, the hierarchical order, is defined as the number of levels of scale displaying a recognised structure [40].

For each level of hierarchy i, two sets of parameters define the configuration:

$$\Gamma^{[i-1]} := \left(A_S^{[i-1]}, \lambda_S^{[i-1]}, \alpha_S^{[i-1]}, \beta_S^{[i-1]}, \vartheta_S^{[i-1]}, h_L^{[i-1]}, \tilde{G}_L^{[i-1]}, \tilde{E}_L^{[i-1]}, \tilde{\nu}_L^{[i-1]}, G_T^{[i-1]}, E_T^{[i-1]}, \nu_T^{[i-1]} \right),$$

$$\Gamma^{[i]} := \left(A_S^{[i]}, \lambda_S^{[i]}, \alpha_S^{[i]}, \beta_S^{[i]}, \vartheta_S^{[i]}, h_L^{[i]}, \tilde{G}_L^{[i]}, \tilde{E}_L^{[i]}, \tilde{\nu}_L^{[i]}, G_T^{[i]}, E_T^{[i]}, \nu_T^{[i]} \right),$$

$$i = 1, 2, ..., n.$$

$$(17)$$

The first, $\Gamma^{[i-1]}$, describes the geometric $(A_S^{[i-1]}, \lambda_S^{[i-1]}, \alpha_S^{[i-1]}, \beta_S^{[i-1]}, \vartheta_S^{[i-1]}, h_L^{[i-1]})$ and mechanical $(\tilde{G}_L^{[i-1]}, \tilde{E}_L^{[i-1]}, \tilde{\nu}_L^{[i-1]}, G_T^{[i-1]}, E_T^{[i-1]}, \nu_T^{[i-1]})$ properties of the interface layer microstructure, the level-[i-1]. The second, $\Gamma^{[i]}$, defines the geometric $(A_S^{[i]}, \lambda_S^{[i]}, \alpha_S^{[i]}, \beta_S^{[i]}, \vartheta_S^{[i]}, h_L^{[i]})$ and mechanical $(\tilde{G}_L^{[i]}, \tilde{E}_L^{[i]}, \tilde{\nu}_L^{[i]}, G_T^{[i]}, E_T^{[i]}, \nu_T^{[i]})$ properties of the underlying large architecture, the level-[i].

By assuming that the size of its microstructure is fine enough to be negligible with respect to the level-[i] structure [40], the interface layer can be treated as a continuum with effective



Figure 6: Viscoelastic hierarchical suture joint: schematic representation and principal geometric parameters in the case of a self-similar triangular configuration.

properties having the same form of Equations (7)–(15). Consequently, for a level-[i] hierarchical suture joint, the effective moduli can be evaluated in the frequency domain by adopting the same approach described in Section 2. Namely, by alternately applying the three loading situations in Equation (5) and by solving the resulting mechanical problem by means of the elastic-viscoelastic correspondence principle in conjunction with the classical mechanics-based analysis presented in [11, 12].

4.2. Effective moduli

Let us focus on the level-[2] suture joint in Figure 6 and, in particular, on the interface layer, the level-[1] structure. As it can be seen, it coincides with the suture joint analysed in Section 2 so that its effective Young's moduli, $E_1^{[1]}$ and $E_2^{[1]}$, shear modulus, $G_{12}^{[1]}$, and Poisson's ratios, $\nu_{12}^{[1]}$ and $\nu_{21}^{[1]}$, can be derived from Equations (7)–(15) by substituting A_S , λ_S , α_S , β_S , ϑ_S , h_L with $A_S^{[1]}$, $\lambda_S^{[1]}$, $\alpha_S^{[1]}$, $\beta_S^{[1]}$, $\vartheta_S^{[1]}$, $h_L^{[1]}$ and G_T , E_T , ν_T , \tilde{G}_L , \tilde{E}_L , $\tilde{\nu}_L$ with the parameters $G_T^{[1]}$, $E_T^{[1]}$, $\nu_T^{[1]}$, $\tilde{G}_L^{[1]}$, $\tilde{E}_L^{[1]}$, $\tilde{\nu}_L^{[1]}$, which specify the geometric and mechanical characteristics of the teeth and of the interface layer in the level-[1] suture joint. Note that, in the starting configuration, i.e., the level-[1], the interface

layer does not have a microstructure, so that $\tilde{G}_L^{[1]} \equiv \tilde{G}_L$, $\tilde{E}_L^{[1]} \equiv \tilde{E}_L$, $\tilde{\nu}_L^{[1]} \equiv \tilde{\nu}_L$.

Having assumed that, at each hierarchical level, the length of scale of the sub-structure is fine enough to be negligible with respect to the super-structure (see Section 4.1), allows us to use $E_1^{[1]}$, $E_2^{[1]}$, $\nu_{12}^{[1]}$, $\nu_{21}^{[1]}$ and $G_{12}^{[1]}$ to obtain the effective moduli of the level-[2] suture joint. This can be done by replacing the geometric parameters A_S , λ_S , α_S , β_S , ϑ_S , h_L in Equations (7)–(15) with $A_S^{[2]}$, $\lambda_S^{[2]}$, $\alpha_S^{[2]}$, $\beta_S^{[2]}$, $\vartheta_S^{[2]}$, $h_L^{[2]}$ and G_T , E_T , ν_T , \tilde{G}_L , \tilde{E}_L , $\tilde{\nu}_L$ with $G_T^{[2]}$, $E_T^{[2]}$, $\nu_T^{[2]}$, $\tilde{G}_L^{[2]} \equiv G_{12}^{[1]}$, $\tilde{E}_L^{[2]} \equiv E_1^{[1]}$, $\tilde{\nu}_L^{[2]} \equiv \nu_{12}^{[1]}$. Note that in the level-[2] suture joint and, in general, in suture joints with hierarchical order i > 1, the interface layer is not isotropic and in approximating it with an equivalent continuum, the effective properties in the longitudinal direction e_1 have been considered [41, 42].

Analogous considerations lead to effective moduli of the level-[n] hierarchical suture joint, derived from Equations (7)–(15) by substituting A_S , λ_S , α_S , β_S , ϑ_S , h_L with $A_S^{[n]}$, $\lambda_S^{[n]}$, $\alpha_S^{[n]}$, $\beta_S^{[n]}$, $\vartheta_S^{[n]}$, $h_L^{[n]}$ and G_T , E_T , ν_T , \tilde{G}_L , \tilde{E}_L , $\tilde{\nu}_L$ with $G_T^{[n]}$, $E_T^{[n]}$, $\nu_T^{[n]}$, $\tilde{G}_L^{[n]} \equiv G_{12}^{[n]}$, $\tilde{E}_L^{[n]} \equiv E_1^{[n-1]}$, $\tilde{\nu}_L^{[n]} \equiv \nu_{12}^{[n-1]}$. It emerges that the effective Young's modulus and related Poisson's ratio in the e_1 direction are given, respectively, by

$$E_1^{[n]} = \left(\frac{\lambda_S^{[n]} - 2h_L^{[n]}}{\lambda_S^{[n]} E_T^{[n]}} + \frac{2h_L^{[n]}}{\lambda_S^{[n]}} \left(\frac{\cos^2 \alpha_S^{[n]} \sin^2 \alpha_S^{[n]}}{\tilde{G}_L^{[n]}} + \frac{\cos^4 \alpha_S^{[n]}}{\tilde{E}_L^{[n]}}\right)\right)^{-1},\tag{18}$$

$$\nu_{12}^{[n]} = -\frac{\lambda_S^{[n]}}{2A_S^{[n]}} \left(\frac{-\frac{\nu_T^{[n]}}{E_T^{[n]}} A_S^{[n]} - \frac{\tilde{\nu}_L^{[n]}}{\tilde{E}_L^{[n]}} \frac{h_L^{[n]}}{\tan \beta_S^{[n]}} + h_L^{[n]} \cos^3 \alpha_S^{[n]} \sin \alpha_S^{[n]} \left(\frac{1}{\tilde{G}_L^{[n]}} + \frac{1}{\tilde{E}_L^{[n]}} \right)}{\frac{\lambda_S^{[n]}}{2 E_T^{[n]}} + h_L^{[n]} \cos^2 \alpha_S^{[n]} \left(\frac{\sin^2 \alpha_S^{[n]}}{\tilde{G}_L^{[n]}} + \frac{\cos^2 \alpha_S^{[n]}}{\tilde{E}_L^{[n]}} \right)} \right),$$
(19)

while the corresponding quantities in the direction e_2 take the form

$$E_{2}^{[n]} = \frac{\lambda_{S}^{[n]} - 2h_{L}^{[n]}}{\lambda_{S}^{[n]}} \left(\frac{\Psi^{[n]}}{E_{T}^{[n]}} + \frac{h_{L}^{[n]} \left(\lambda_{S}^{[n]} - 2h_{L}^{[n]}\right)}{2\left(A_{S}^{[n]}\right)^{2}} \left(\frac{\cos^{4}\alpha_{S}^{[n]}}{\tilde{G}_{L}^{[n]}} + \frac{\cos^{2}\alpha_{S}^{[n]}\sin^{2}\alpha_{S}^{[n]}}{\tilde{E}_{L}^{[n]}} \right) \right)^{-1}, \quad (20)$$

$$\nu_{21}^{[n]} = \frac{\left(\frac{\nu_T^{[n]}A_S^{[n]}}{E_T^{[n]}} + \frac{\tilde{\nu}_L^{[n]}h_L^{[n]}}{\tilde{E}_L^{[n]}\tan\beta_S^{[n]}} - h_L^{[n]}\cos^3\alpha_S^{[n]}\sin\alpha_S^{[n]}\left(\frac{1}{\tilde{G}_L^{[n]}} + \frac{1}{\tilde{E}_L^{[n]}}\right)\right)\left(1 + \frac{2h_L^{[n]}}{\lambda_S^{[n]}}\left(\Phi_1^{[n]} - 1\right)\right)}{\left(\frac{h_L^{[n]}}{2A_S^{[n]}}\frac{\Phi_1^{[n]}}{E_T^{[n]}} + \frac{A_S^{[n]}}{\left(\lambda_S^{[n]} - 2h_L^{[n]}\right)}\frac{\Psi^{[n]}}{E_T^{[n]}}\right)\left(\lambda_S^{[n]} + 2h_L^{[n]}\Phi_1^{[n]}\right)},$$
(21)

with

$$\Psi^{[n]} := \begin{cases} 1 & \text{triangular} \\ 2/3 & \text{rectangular} \\ 2 - \frac{1}{\psi_{\alpha\beta}^{[n]}} + \frac{\left(1 - \psi_{\alpha\beta}^{[n]}\right)^2}{2\psi_{\alpha\beta}^{[n]} 2} \ln\left(\frac{1 + \psi_{\alpha\beta}^{[n]}}{1 - \psi_{\alpha\beta}^{[n]}}\right) & \text{trapezoidal, anti-trapezoidal,} \\ \psi_{\alpha\beta}^{[n]} := \frac{\tan \alpha_S^{[n]}}{\tan \beta_S^{[n]}} \end{cases}$$
(23)

and

$$\Phi_1^{[n]} := E_T^{[n]} \left(\frac{\cos^2 \alpha_S^{[n]} \sin^2 \alpha_S^{[n]}}{\tilde{G}_L^{[n]}} + \frac{\cos^4 \alpha_S^{[n]}}{\tilde{E}_L^{[n]}} \right).$$
(24)

Finally, the effective shear modulus $G_{12}^{[n]}$ is given by

$$G_{12}^{[n]} = \frac{\left(\lambda_S^{[n]} - 2h_L^{[n]}\right)\Phi_2^{[n]}/\lambda_S^{[n]}}{\left(\frac{\Phi_2^{[n]}}{4\Phi_3^{[n]}}\left(3 + \frac{5\Phi_3^{[n]}}{4E_T^{[n]}\tan^2\beta_S^{[n]}}\right) + \frac{\lambda_S^{[n]}\tan\alpha_S^{[n]}}{2A_S^{[n]}}\left(\frac{2h_L^{[n]}}{\lambda_S^{[n]}\psi_{\alpha\beta}^{[n]}} + \frac{3\tilde{G}_L^{[n]}}{2\Phi_3^{[n]}}\left(1 + \frac{3\Phi_3^{[n]}}{4E_T^{[n]}\tan^2\beta_S^{[n]}}\right)\right)\right)}$$
(25)

where, to simplify the notation,

$$\Phi_2^{[n]} := \tilde{G}_L^{[n]} \tan^2 \alpha_S^{[n]} + \tilde{E}_L^{[n]}, \qquad \Phi_3^{[n]} := \frac{G_T^{[n]} \left(\lambda_S^{[n]} - 2h_L^{[n]}\right) + 2\tilde{G}_L^{[n]} h_L^{[n]}}{\lambda_S^{[n]}}.$$
(26)

As in the non-hierarchical case, the effective moduli listed above are complex quantities and, again, the loss tangent can be defined (see Section 2.2)

$$\tan \delta_1^{[n]} \equiv \frac{(E_1^{[n]})''}{(E_1^{[n]})'}, \quad \tan \delta_2^{[n]} \equiv \frac{(E_2^{[n]})''}{(E_2^{[n]})'}, \quad \tan \delta_{12}^{[n]} \equiv \frac{(G_{12}^{[n]})''}{(G_{12}^{[n]})'}, \quad (27)$$

being, on order, $(\cdot)^{'}$ and $(\cdot)^{''}$ the storage and loss moduli of the level-[n] hierarchical suture joint.

5. Parametric analysis and optimal values

From Equations (18)-(26) it clearly emerges that the effective moduli of a level-[n] hierarchical suture joint are influenced not only by the geometric and mechanical properties of the two phases at level-[n], but also by the corresponding characteristics of the smaller length scales. Motivated by the findings in Section 3, where even an auxetic behavior can be achieved for particular values of the tooth tip angle, this section aims at understanding how the suture characteristics affect the effective moduli in the case of structural hierarchy. The analysis deals with a viscoelastic hierarchical suture joint having up to 11 levels of hierarchy and, considering its abundant presence in biology, a self-similar triangular configuration with a constant value of teeth volume fraction at all levels. A self-similar entity, in particular, exhibits a statistically similar characteristic if examined both locally, at the level of individual components, and globally, at the level of the whole system [43]. It can be thus said that the same general trait is independent of the scale at which the observation is made [44] and this assumption renders the problem more treatable.



Figure 7: Normalised effective stiffness of viscoelastic hierarchical suture joints vs number of levels : (a) $(E_1^{[i]})'/E_T$, (b) $(E_2^{[i]})'/E_T$, (c) $(G_{12}^{[i]})'/E_T$.

The starting element of the hierarchical arrangement considered, i.e., the level-[1] structure, has the same characteristics of the suture joint analysed in Section 3. Namely, wavelength $\lambda_S = 15$

mm, amplitude $A_S = 15$ mm, mechanical properties of the teeth given by $E_T = 6$ GPa, $G_T = 2.3$ GPa, $\nu_T = 0.3$, and mechanical properties of the interface layer provided by $E_L = 10^{-2}E_T$, $G_L = 10^{-2}G_T$, $\nu_L = 0.3$ and $\eta = \xi = 5$ Pa · s. Finally, a teeth volume fraction of $\phi_T = 0.75$ is selected, value that is constant at all levels.



Figure 8: Viscoelastic hierarchical suture joints: loss factors vs number of levels: (a) $\tan \delta_1^{[i]}$, (b) $\tan \delta_2^{[i]}$, (c) $\tan \delta_{12}^{[i]}$.

Results are summarised in Figures 7, 8, where the normalised effective stiffness $(E_1^{[i]})'/E_T$, $(E_2^{[i]})'/E_T$, $(G_{12}^{[i]})'/E_T$ and loss tangent $\tan \delta_1^{[i]}$, $\tan \delta_2^{[i]}$, $\tan \delta_{12}^{[i]}$ are plotted versus the number of levels, and in Figure 9, where the effective Poisson's ratios $\nu_{12}^{[i]}$, $\nu_{21}^{[i]}$ are plotted versus the tooth tip angle $\alpha_S = \beta_S$. Note that, according to the assumption of self-similarity, α_S is constant at all levels.

As illustrated in Figure 7, increasing the number of hierarchical levels leads to an increase in the effective stiffness, which is more significant up to a number of orders n < 5. Conversely, when $n \ge 5$, the curves reach a sort of plateau zone and increasing the number of levels provides a very small increase, independently of the value of the tooth tip angle α_s .

A different trend can be observed in Figure 8 where, in the case of the loss tangent, increasing the number of hierarchical levels leads to a significant improvement only for particular values of the parameter α_S . For example, if we focus on $\tan \delta_2^{[i]}$ it emerges that hierarchy is detrimental for $\alpha_S = 30^\circ, 40^\circ$ while, for $\alpha_S = 60^\circ$, adding hierarchy leads to an improvement up to the second level, which coincides with the optimum level of this particular configuration (Fig. 8b).

Regarding the Poisson's ratios in Figure 9, only the curves corresponding to the first 5 levels of hierarchy are considered, allowing us to provide a more clear representation. Also, to deeply

investigate the possibility to obtain auxeticity in the hierarchical context, three different values of teeth volume fraction are examined: $\phi_T = 0.6, 0.7, 0.8$. Generally, an auxetic response appears for all the investigated cases, even if a large variability in the values of $\nu_{12}^{[i]}$ and $\nu_{21}^{[i]}$ can be obtained simply by modifying the angle α_S and the number of hierarchical levels n. This aspect is well illustrated in Figures 9c, 9f where, for $\phi_T = 0.8$, auxeticity emerges only for n = 1, 2 while, when n = 3, 4, 5, the effective Poisson's ratios are always positive. Conversely, $\phi_T = 0.6$ (Figs. 9a, 9d) and $\phi_T = 0.7$ (Figs. 9b, 9e), lead to an auxetic response independently of the hierarchical order considered. By comparing Figures 9a, 9d and Figures 9b, 9e, we also observe that increasing the teeth volume fraction from $\phi_T = 0.6$ to $\phi_T = 0.7$ reduces the minimum value of $\nu_{12}^{[i]}$ and $\nu_{21}^{[i]}$ that can be achieved.

From these results, it can be said that there exist a synergy of hierarchy, geometric features and material heterogeneity in obtaining auxeticity.



Figure 9: The influence of the tooth tip angle α_S into the effective Poisson's ratios $\nu_{12}^{[i]}$ (first row) and $\nu_{21}^{[i]}$ (second row) in the case of (a), (d) $\phi_T = 0.6$; (b), (e) $\phi_T = 0.7$; (c), (f) $\phi_T = 0.8$.

6. Different levels with different geometry

To further investigate how the suture parameters can be optimized to improve its mechanical response in the hierarchical context, this section deals with a four-level viscoelastic hierarchical suture joint having different waveforms at each level: triangular, rectangular, trapezoidal and anti-trapezoidal. Such geometries are obtained, respectively, for $\alpha_S/\beta_S = 1$, $\alpha_S/\beta_S = 0$, $\alpha_S/\beta_S = 0.6$ and $\alpha_S/\beta_S = -0.6$ (Fig. 1).

In all the examined configurations (Table 1), the starting element, i.e., the level-[1], has teeth with Young's modulus $E_T = 6$ GPa, shear modulus $G_T = 2.3$ GPa, Poisson's ratio $\nu_T = 0.3$ and viscoelastic interface layer with Young's modulus $E_L = 10^{-1}E_T$, shear modulus $G_L = 10^{-1}G_T$, Poisson's ratio $\nu_L = 0.3$ and viscosity coefficients $\eta = \xi = 50$ Pa · s. Also, a teeth volume fraction of $\phi_T = 0.75$ is selected, value that is constant at all levels.

To keep the paper concise, results are reported for the first four configurations (TRI1, TRI2, TRI3, TRI4) only. For additional details, the interested reader is referred to the Supplementary Material. In terms of effective stiffness $(E_1^{[4]})'/E_T$, $(E_2^{[4]})'/E_T$ and $(G_{12}^{[4]})'/E_T$, we observe a decrease provided by increasing the tooth tip angle β_S (Fig. 10).



Figure 10: Influence of the tooth tip angle β_S into the effective stiffness of hierarchical sutures having a triangular profile at the first level: (a) $(E_1^{[4]})'/E_T$, (b) $(E_2^{[4]})'/E_T$, (c) $(G_{12}^{[4]})'/E_T$

As expected, different results emerge from the examined configurations. For example, having a rectangular (TRI2, TRI5) profile at the fourth level yields higher values of $(E_1^{[4]})'/E_T$ (Fig. 10a) while the arrangement in TRI6 experiences superior $(E_2^{[4]})'/E_T$ (Fig. 10b). Figure 10c reveals the

	level-[1]	level-[2]	level-[3]	level-[4]
TRI1	triangular	trapezoidal	rectangular	anti-trapezoidal
TRI2	triangular	trapezoidal	anti-trapezoidal	rectangular
TRI3	triangular	rectangular	trapezoidal	anti-trapezoidal
TRI4	triangular	rectangular	anti-trapezoidal	trapezoidal
TRI5	triangular	anti-trapezoidal	trapezoidal	rectangular
TRI6	triangular	anti-trapezoidal	rectangular	trapezoidal
TRI7	triangular	triangular	triangular	triangular
TRA1	trapezoidal	triangular	rectangular	anti-trapezoidal
TRA2	trapezoidal	triangular	anti-trapezoidal	rectangular
TRA3	trapezoidal	rectangular	${ m triangular}$	anti-trapezoidal
TRA4	trapezoidal	rectangular	anti-trapezoidal	triangular
TRA5	trapezoidal	anti-trapezoidal	triangular	rectangular
TRA6	trapezoidal	anti-trapezoidal	rectangular	triangular
TRA7	trapezoidal	trapezoidal	trapezoidal	trapezoidal
REC1	rectangular	triangular	trapezoidal	anti-trapezoidal
REC2	rectangular	triangular	anti-trapezoidal	trapezoidal
REC3	rectangular	trapezoidal	triangular	anti-trapezoidal
REC4	rectangular	trapezoidal	anti-trapezoidal	triangular
REC5	rectangular	anti-trapezoidal	triangular	trapezoidal
REC6	rectangular	anti-trapezoidal	trapezoidal	triangular
REC7	rectangular	rectangular	rectangular	rectangular
ANTI1	anti-trapezoidal	triangular	trapezoidal	rectangular
ANTI2	anti-trapezoidal	triangular	rectangular	trapezoidal
ANTI3	anti-trapezoidal	trapezoidal	triangular	rectangular
ANTI4	anti-trapezoidal	trapezoidal	rectangular	triangular
ANTI5	anti-trapezoidal	rectangular	triangular	trapezoidal
ANTI6	anti-trapezoidal	rectangular	trapezoidal	triangular
ANTI7	anti-trapezoidal	anti-trapezoidal	anti-trapezoidal	anti-trapezoidal

Table 1: Different levels with different geometry: hierarchical configurations examined

existence of a particular value of $\beta_S, \, \beta_S^* \approx 46^\circ$, such that

$$\left(\frac{\left(G_{12}^{[4]}\right)'}{E_T}\right)_{\text{TRI6, TRI4}} > \left(\frac{\left(G_{12}^{[4]}\right)'}{E_T}\right)_{\text{TRI1, TRI3}} \text{ for } \beta_S < \beta_S^*,$$

$$\left(\frac{\left(G_{12}^{[4]}\right)'}{E_T}\right)_{\text{TRI6, TRI4}} < \left(\frac{\left(G_{12}^{[4]}\right)'}{E_T}\right)_{\text{TRI1, TRI3}} \text{ for } \beta_S > \beta_S^*.$$
(28)

That is to say, for small values of β_S , $\beta_S < \beta_S^*$, the hierarchical arrangements TRI6, TRI4 (with a trapezoidal geometry at the fourth level) are stiffer than TRI1, TRI3 (with an anti-trapezoidal profile at the fourth level) while, for $\beta_S > \beta_S^*$, an opposite trend appears.



Figure 11: Influence of the tooth tip angle β_S into the effective Poisson's ratios of hierarchical sutures having a triangular profile at the first level: (a) $\nu_{12}^{[4]}$, (b) $\nu_{21}^{[4]}$.

Figures 11 suggests that the suture profile at different levels is an important parameter to achieve an auxetic response. For example, if we consider Figure 11a, we observe that auxeticity can be obtained only with a triangular (TRI7) or trapezoidal (TRI5, TRI4) geometry at level-[4].

A common feature of Figure 12 is that, in all the examined configurations, for the loss tangent $\tan \delta_1^{[4]}$, $\tan \delta_2^{[4]}$, $\tan \delta_1^{[4]}$ the effect of β_S is generally minimal for $\beta_S < 40^\circ$. However, different levels of dissipation are provided by the different configurations examined. For example, TRI7 (triangular profile at all levels) experiences higher values of $\tan \delta_1^{[4]}$ (Fig. 12a) while, in terms of $\tan \delta_{12}^{[4]}$, higher values are observed for TRI1, TRI3 (with an anti-trapezoidal waveform at level-[4]).

In the context of hierarchical suture joints, these results reveal the possibility to improve the mechanical response, in terms of stiffness, auxeticity and dissipation levels, by adopting a sort of 'mix and match' approach. That is to say, an approach in which different levels have different geometric features.



Figure 12: Influence of the tooth tip angle β_S into the loss tangent of hierarchical sutures having a triangular profile at the first level: (a) $\tan \delta_1^{[4]}$, (b) $\tan \delta_2^{[4]}$ (c) $\tan \delta_{12}^{[4]}$.

7. An application of the theory to the sinusoidal patterned sutures in biology

As a real-case example, in this section we apply the theoretical concepts presented in Section 2 to obtain the effective stiffness of sinusoidal sutures, typical of many biological systems (Figure 13). Examples include the red-bellied woodpecker beak [3] and the common millet seedcoat [31], where the sinusoidal geometry has been demonstrated to be responsible for increasing energy dissipation and impact resistance, sea urchins, where the sinusoidal architecture has been developed to force cracks to follow a specific pattern, and the cranium of different biological species like monkeys [45], goats [8], lizards [49], dinosaurs [50] and humans [51, 9]. Cranial sutures, in particular, provide the cranial bone flexibility for growth and movement, alleviate stress during mastication and impact phenomena, and greatly enhance the overall stiffness and strength [9].

Similarly to Section 2, in deriving the effective stiffness of the sinusoidal suture, our attention is focused on the interdigitating area and, due to the periodicity of the configuration, only the Representative Volume Element is considered (Figure 14). From a geometrical point of view, its configuration is described by the two parameters ϕ_t , volume fraction of the teeth, and β_s , tooth tip angle, given respectively by [31]

$$\phi_T = 1 - \frac{\mathcal{L}_S h_L}{\lambda_S \left(2A_S + h_L\right)}, \qquad \beta_S = \tan^{-1}\left(\frac{\lambda_S}{4A_S}\right), \tag{30}$$



Figure 13: Sinusoidal sutures in biology: (a) red-bellied woodpecker beak (modified from [3], [46]), (b) sagittal sutures of the human cranium (modified from [47], [48]), (c) marine threespine stickleback (modified from [18]).

with A_S and λ_S , in turn, the amplitude and wavelength of the suture, h_L the thickness of the interface layer,

$$\mathcal{L}_{\mathcal{S}} = \int_{0}^{\lambda_{S}} \sqrt{1 + \left(\frac{d y_{S}(x)}{dx}\right)^{2} dx}$$
(31)

the arc length of the sinusoidal wave profile [52]

$$y_S(x) = A_S \left[1 + \sin\left(2\pi \left(x - \frac{\lambda_S}{4}\right) \frac{1}{\lambda_S}\right) \right]$$
(32)

within one wavelength.

The simplifying hypothesis of Section 2, i.e., teeth and interface layer homogeneous and perfectly bonded at the interfaces, still apply. Again, a linear elastic mechanical behavior is assumed for the teeth and a viscoelastic response described by the Kelvin-Voigt model is considered for the interface layer.

To derive the effective stiffness of the sinusoidal suture, the same methodology of Section 2 can be adopted. That is, by alternately prescribing the loading conditions in Equation (5) and



Figure 14: Schematic representation of a sinusoidal suture joint indicating the most relevant geometric parameters.

solving the corresponding problem by using the mechanics-based analysis of [11, 12], together with the elastic-viscoelastic correspondence principle. In this example, for simplicity, only the effective Young's moduli in the e_1 and e_2 direction are considered. They are given, respectively, by

$$E_1 = \left[(1 - \phi_T) \left(\frac{\cos^2 \beta_S \sin^2 \beta_S}{\tilde{G}_L} + \frac{\cos^4 \beta_S}{\tilde{E}_L} \right) + \frac{\phi_T}{E_T} \right]^{-1} \equiv E_1' + i E_1''$$
(33)

and

$$E_{2} = \phi_{T}^{2} \left[\frac{\phi_{T}}{E_{T}} + (1 - \phi_{T}) \left(\frac{\cos^{2}\beta_{S} \sin^{2}\beta_{S}}{\tilde{G}_{L}} + \frac{\sin^{4}\beta_{S}}{\tilde{E}_{L}} \right) \right]^{-1} \equiv E_{2}' + iE_{2}'',$$
(34)

with E_T the Young's modulus of the teeth, \tilde{G}_L and \tilde{E}_L the frequency-dependent shear modulus and Young's modulus of the interface layer (cf. Section 2.2), (·)['] and (·)^{''}, respectively, the storage and loss moduli of the suture, such that their ratio provides the loss tangent $\tan \delta_1 \equiv \frac{E_1''}{E_1'}$ and $\tan \delta_2 \equiv \frac{E_2''}{E_2'}$. From Equations (33),(34) it clearly emerges that the effective moduli of a sinusoidal suture are

From Equations (33),(34) it clearly emerges that the effective moduli of a sinusoidal suture are strongly affected by the suture waviness $\chi := \frac{2A_S}{\lambda_S}$, i.e., the ratio between the suture height and the suture wavelength, via the parameter β_S . In biology, the values of χ vary between 1, in the case of the woodpecker beak [3], and approximately 4 in the case of the cranium [51]. This leads to

higher values of effective stiffness, as illustrated in Figure 15, where E'_1/E_T and E'_2/E_T are plotted against the parameter χ . Figure 15, in particular, is based on the data for the human cranium [51]: tooth material with $E_T=6$ GPa, $\nu_T=0.3$, $G_T=2.3$ GPa; interface layer with $E_L=80$ MPa, $\nu_L=$ 0.4, $G_L=26.8$ MPa and $h_L=0.2$ mm, providing $\phi_T=0.85$; $A_S=5.4$ mm and $\lambda_S=2.7$ mm leading to $\chi = 4$ and $\beta_S \approx 7.1^{\circ}$. Finally, we have assumed $\eta = \xi = 5Pa$. From Figure 15 it can be said that this particular configuration is the optimal one to obtain higher values of stiffness since an increase in χ , provided by fixing λ_S and varying A_S , does not produce significant additional benefits. Conversely, a decrease in the effective stiffness is observed by decreasing χ .



Figure 15: An example from biology: the sinusoidal sutures of the cranium. Influence of the waviness χ in the normalised effective stiffness: (a) Young's modulus E'_1/E_T , (b) Young's modulus E'_2/E_T .

A different trend emerges for the loss tangent illustrated in Figure 16. As it can be seen, to small values of χ , i.e., $\chi \ll 1.5$, correspond high values of $\tan \delta_1$ and $\tan \delta_2$ while a sort of plateau can be observed for $\chi \gg 2.5$. As expected, a very high dissipation level is observed for $\chi = 1$, value that coincides with the waviness of the red-bellied woodpecker beak. It can be thus said that while the waviness of the cranium is optimized to obtain high stiffness, that of the woodpecker beak, due to its particular function, is optimized to obtain high levels of dissipation.

Even if a detailed analysis of the sinusoidal sutures in biology and, in particular, of the sinu-



Figure 16: The sinusoidal sutures of the cranium: influence of the waviness χ in the in the loss tangent (a) $\tan \delta_1$, (b) $\tan \delta_2$.

soidal sutures of the cranium, goes beyond the aim of our work (more sophisticated techniques are required), these results confirm two important aspects. The first, is the great advantage offered by suture joints, the second is that biological systems are efficiently designed to obtain an optimal mechanical behavior.

8. Conclusions

This paper deals with the analysis of viscoelastic suture joints having a general trapezoidal waveform.

Firstly, based on the elastic-viscoelastic correspondence principle, a theoretical model leading to closed-form expressions for the effective properties (Young's moduli, shear modulus, Poisson's ratios) and loss factors is presented. A strong influence of the suture geometric and mechanical characteristics on the effective behavior emerges and the results of a parametric analysis to investigate this aspect are reported. By carefully tailoring the suture parameters, i.e., geometric profile, tooth tip angle and teeth volume fraction, not only an improvement in the effective stiffness and loss moduli, but also an auxetic response, can be obtained.

Viscoelastic hierarchical suture joints, obtained by adding some levels of hierarchy into the starting system, are also investigated. The application of the elastic-viscoelastic correspondence

principle leads to explicit relations for the effective Young's modulus, shear modulus, Poisson's ratios and loss factors. These quantities are closely related to the suture characteristics at all levels and the example of a self-similar hierarchical structure with eleven orders of hierarchy clarify this. The analysis reveals that introducing hierarchy improves the effective moduli and loss factors of the non-hierarchical counterpart. Optimal levels of hierarchy can be identified and, similarly to the non-hierarchical case, auxeticity appears for certain values of the suture tooth tip angle and volume fraction. The example of a four-level viscoelastic suture joint having different waveforms at each level is then presented. This sort of 'mix and match' approach illustrates how the geometric characteristics of the suture at different levels are an important parameter in terms of effective stiffness and dissipation level and suggests a possible design concept for bioinspired materials based on structural hierarchy, material mixing and geometric 'mix and match' approach. An extension of the theory to the sinusoidal patterned sutures in biology is also included. By considering the cranial sutures as a real-case example, the analysis confirms the functional significance of biological sutures, as well as the efficiency of biological systems in obtaining an optimal mechanical behavior. In particular, if compared to the sinusoidal sutures of the woodpecker beak, it emerges that the configuration of cranial sutures is optimized to obtain high stiffness, while that of the woodpecker beak for providing high energy dissipation levels. Compared to the previous works, this comprehensive study is the first to explore the mechanical behavior of suture joints with the consideration of both hierarchy and viscoelasticity.

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Highlights

- this paper describes the mathematical formulation and modelling technique leading to explicit expressions for the effective properties of viscoelastic suture joints with a general trapezoidal waveform;
- a strong influence of the suture geometric and mechanical characteristics on the effective properties emerges and a parametric analysis reveals that an auxetic behavior can be obtained simply by tailoring the suture parameters;
- the effects of adding hierarchy into the above system are also explored and closed-form relations for the effective moduli are derived. Optimal levels of hierarchy can be identified and, similarly to the non-hierarchical case, an auxetic behavior emerges for particular values of the suture parameters.
- An extension of the theory to the sinusoidal sutures in biology is also reported. By considering the examples of the cranium and the woodpecker beak, it emerges that the first is optimized to obtain high stiffness, while the second to obtain high energy dissipation levels.

Author Agreement Statement

The authors declare that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere. We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed.

We further confirm that the order of authors listed in the manuscript has been approved by all of us. We understand that the Corresponding Author is the sole contact for the Editorial process. He/she is responsible for communicating with the other authors about progress, submissions of revisions and final approval of proofs.

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Declaration of interests

The authors of the manuscript, F. Ongaro and N. Pugno, declare:

X The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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