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Fracture Initiation at Re-entrant Corners: Experiments and Finite Fracture Mechanics Predictions

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ABSTRACT: *In this paper the results of a series of three-point bending experimental tests for specimens made of polystyrene and containing re-entrant corners are firstly presented. Tests involved different notch angles, different notch depths and finally different dimensions of the samples. Because of the structural brittleness, the generalized stress intensity factor was expected to be the governing failure parameter. Results are compared with the predictions provided by a fracture criterion recently introduced in the framework of Finite Fracture Mechanics: fracture is assumed to propagate by finite steps, whose length is determined by the contemporaneous fulfilment of energy balance and stress requirements. This criterion allows us to achieve the expression of the generalized fracture toughness as a function of the tensile strength, the fracture toughness and the notch opening angle.*

KEY WORDS: *Three-point bending tests; V-notches; Fracture toughness; Tensile strength; Finite Fracture Mechanics*

1. Introduction

Criteria assuming that failure of quasi-brittle materials is affected by the stresses acting at a finite distance from the crack tip are widely used inside the Scientific Community. These approaches can be grouped together under the general term of Theory of Critical Distances (Taylor, 2004), in which linear-elastic analysis is combined with a material-dependent length. The most common criterion assumes as a critical parameter the average stress over a characteristic material length Δ ahead of the crack tip. However, this kind of criteria disregards energy balance considerations, which, as well known, are the basis of the Linear Elastic Fracture Mechanics.

On the other hand, the novelty of the criterion used in the present paper relies on the fulfilment of the energy balance, under the assumption of a finite crack extension: failure is achieved whenever there is a segment of length Δ ahead of the notch tip over which the stress resultant is equal to $\sigma_u\Delta$, and, contemporarily, the energy available for that crack extension is equal to $\mathcal{G}_F\Delta$. σ_u and $\mathcal{G}_F$ are the material tensile strength and fracture energy, respectively. Differently from the average stress criterion, the length Δ is no more a material constant but a structural parameter, thus able to take the interaction between the finite crack extension and the geometry of the specimen into account. Henceforth we refer to this criterion as the coupled Finite Fracture Mechanics (FFM) criterion (Cornetti *et al.*, 2006).

In the next Section a set of experimental data obtained recording the failure loads of V-notched specimens is presented. This kind of tests is similar to others existing in the literature (Carpinteri, 1987; Seweryn, 1994). In Section 3 results will be compared with the theoretical predictions obtained by applying the coupled FFM criterion to the present geometry, aiming to check the validity of this approach.

2. Notched three-point bending tests

A series of tests with notched three-point bending specimens made of polystyrene was carried out.

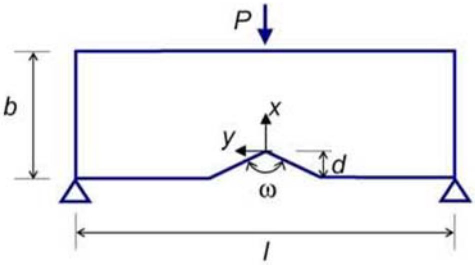

Figure 1. *TPB test of a V-notched specimen*

Data about polystyrene specimens are not common in the literature. Although brittle for usual laboratory sizes, polystyrene is less brittle than PMMA which is considered as the archetype of brittle polymers and for which, consequently, a large amount of data is already available (see e.g. Carpinteri, 1987; Dunn *et al.*, 1997).

The specimen geometry is as in figure 1. Three kinds of test were performed by varying the notch angle ω (*test 1*), the notch depth d (*test* 2) and the specimen dimensions l, b and d in a proportional way (*test 3*), respectively.

Notched flexure specimens were initially machined from polystyrene with the following dimensions: the thickness t was equal to 3.7 mm (which is enough to get plain strain conditions); the length l and the height b were equal to 76 and 18 mm. The notch wedge angles were ω = 60°, 120° and 150°.

A notch depth of d=1.8 mm (d/b=1/10) was firstly machined for each of the three notch angles mentioned (*test 1*). Then, for the 120°-notch samples, three other notch depths were machined: d = 0.2, 0.6 and 10.8 mm. These result in d/b ratios of 1/90, 1/30 and 6/10, respectively (*test 2*). Finally, keeping the notch angle ω fixed to 120° and the relative notch depth d/b to 1/10, all the specimen dimensions (except the thickness t) were decreased by a factor of 5. This corresponds to: l = 15.2mm, b = 3.6mm, d = 0.36mm (*test 3*, figure 2).

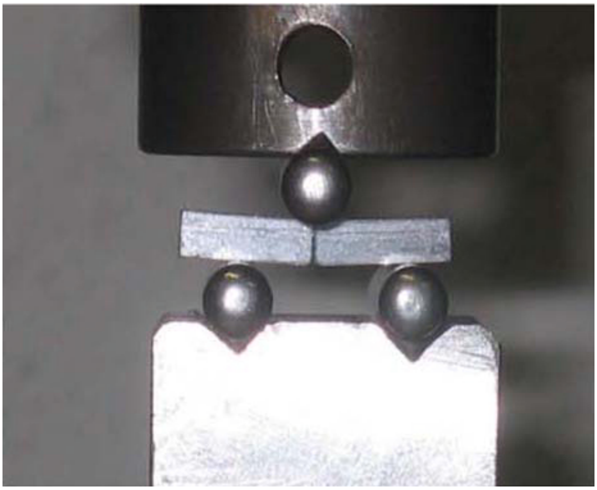

Figure 2. *Test 3: TPB test of a polystyrene beam*

Five identical specimens were tested for each of the seven geometries contemplated. Moreover, five plain specimens were tested to obtain the tensile strength value σ_u, which was found equal to 70.6 MPa, i.e. a total of forty specimens was tested. Anyway, beyond the tensile strength, also the fracture toughness is required to predict the failure load according to the coupled FFM criterion (see Section 3). This was not obtained experimentally. Thus a best fit procedure exploiting the data of all the notched specimens was used to get its value (an analogous procedure was used in Seweryn (94)): K_{Ic}=2.23 MPa $\sqrt{}$ m.

Test 1 and *test 2* were carried out at a strain rate of 10 mm/min. On the other hand, a different machine was used to carry out *test 3* because of the lower distance between the two supports: specimens were tested under a strain rate of 1mm/min.

Specimens were checked with a measuring microscope and the notch root radius was kept very small for each of them (the maximum value was 0.02 mm for the geometry with ω = 60°) : it has been reasonably assumed that the root radius did not affect the experimental results and, therefore, the notch can be considered sharp.

To what concerns *test 1*, note that the specimen dimensions are similar to those used in Dunn *et al.* (1997). They carried out three-point bending tests on PMMA samples with notch opening angles equal to 60°, 90° and 120°.

On the other hand, cases considered in *test 2* are interesting since they represent the limit cases for the criteria based on the generalized fracture toughness to work, i.e. very small and very large notch depths.

Eventually, *test 3* was carried out to catch the well known size scale effects (Carpinteri, 1987), i.e. embrittlement of the structural response as the size increases.

2.1. *Results and physical considerations*

Critical values of the load under which crack begins to propagate from the notch vertex, recorded on the bending testing machine, are given in table 1, table 2 and table 3 for *test 1*, *test 2* and *test 3*, respectively. The divergence of the standard deviation over the average is relatively small in all the cases, the maximum deviation being 6.3%. This highlights the good repeatability in the tests and the small scatter experienced.

It is observed that the critical load increases as the notch angle increases and/or as the notch depth increases. Anyway, there is not a significant difference between the failure load for the 60°-notch samples and the 120°-notch samples. This interesting result will be confirmed by theoretical analysis in the next sections.

The polystyrene fracture was of brittle character and no plastic strains were observed in *test 1* and *2*: all the specimens broke at the defect. On the contrary, a more ductile behaviour was observed during *test 3*: the two pieces into which specimens used to shatter, always remained attached after failure, revealing the presence of a plastic hinge. Anyway this is not so surprising if we refer to the brittleness number s:

$$s = \frac{K_{Ic}}{\sigma_u \sqrt{b}} \tag{1}$$

The brittleness number s is a non-dimensional quantity, introduced by Carpinteri (1981), which describes in a unitary and synthetic manner the embrittlement of the structural response as the size increases. Brittle structural behaviours are generally expected for low brittleness numbers. In the next sections, predictions will refer to a brittleness number s = 0.236 for samples of *test 1* and *test 2* and s = 0.503 for samples of *test 3*.

ω	1	2	3	4	5	Avg.	St.Dev./Avg.
60°	332	323	319.5	345	293.2	322.5	0.06
120°	317.7	342.2	324.7	320.2	308.7	322.7	0.04
150°	369.2	327	327	345.5	340.5	341.8	0.05

Table 1. *Results test 1: Failure load and standard deviation* [N]

d/b	1	2	3	4	5	Avg.	St.Dev./Avg.
1/90	592	520	507.7	518.5	526.5	533	0.06
1/30	456	453	414	447	445	443	0.04
1/10	317.7	342.2	324.7	320.2	308.7	322.7	0.04
6/10	60	60.4	57.9	62.4	63.3	60.8	0.03

Table 2. *Results test 2: Failure load and standard deviation* [N]

Scaling factor	1	2	3	4	5	Avg.	St.Dev./Avg.
1	317.7	342.2	324.7	320.2	308.7	322.7	0.04
0.2	108	109	106	113	102	107.6	0.03

Table 3. *Results test 3: Failure load and standard deviation* [N]

3. Coupled FFM criterion

According to the coupled FFM criterion, failure takes place when both these equations are satisfied:

$$\begin{cases} \int_0^{\Delta} \sigma_y(x)\,\mathrm{d}x = \sigma_u \Delta \\ \int_0^{\Delta} K_I^2(a)\,\mathrm{d}a = K_{Ic}^2 \Delta \end{cases} \qquad [2]$$

where $\sigma_y(x)$ is the stress field ahead of the crack tip, a the crack length, $K_I(a)$ the stress intensity factor, Δ the finite crack extension and K_{Ic} the fracture toughness. Equation [2] represents a system of two equations in the two unknowns: σ_f i.e. the failure load (implicitly embedded in the functions $\sigma_y(x)$ and $K_I(a)$), and Δ, i.e. the crack extension.

Let us consider the specimen geometry of figure 1. By means of dimensional analysis it is possible to write the generalized SIF as:

$$K_I^* = \frac{Pl}{tb^{1+\lambda}} f\left(\frac{l}{b}, \frac{d}{b}, \omega\right) \qquad [3]$$

where f is the so-called shape function and λ is an exponent which depends on the angle ω according to the classical Williams' analysis: $\lambda=\lambda(\omega)$. In order to compute the values of the shape function, a FEA was performed by using the LUSAS ® code for each tested geometry. In critical conditions, equation [3] becomes:

$$K_{Ic}^* = \frac{P_{cr}^{\omega} l}{tb^{1+\lambda}} f\left(\frac{d}{b}, \omega\right), \qquad [4]$$

and, if $\omega = 180°$, yields:

$$\sigma_u = \frac{P_{cr}^{\pi} l}{tb^2} f\left(\frac{d}{b}, \pi\right) \qquad [5]$$

where K_{Ic}^* is the generalized toughness. For the sake of simplicity, the dependence of the shape function on the specimen slenderness l/b is not given explicitly, since it was kept constant in all the tests. Moreover, note that the value of f corresponding to $\omega = 180°$ can be found in structural mechanics classical books (it provides the maximum normal stress for an un-notched TPB specimen). P_{cr}^{ω} and P_{cr}^{π} are the failure load for a notch opening angle equal to ω and to 180°, respectively.

Now the coupled FFM criterion is applied to determine the values of the generalized toughness as a function of the notch opening angle ω. To this purpose, it is necessary to find the expressions for the stress field $\sigma_y(x)$ and for the SIF $K_I(a)$ to be inserted into the system [2]. If the crack advancement Δ is small enough with respect to the other geometrical quantities, the stress field $\sigma_y(x)$ con be sufficiently well described by its asymptotic expansion, according, again, to the well-known Williams' analysis. On the other hand, the SIF $K_I(a)$ can be obtained from the weight functions providing the SIF for a crack at a V-notch tip of an infinite plate loaded by a pair of forces acting on the crack lips. The principle of effects superposition is then invoked. For details, see (Carpinteri *et al.*, 2007).

Solving system [2] yields the value of the finite crack extension:

$$\Delta = \frac{2}{\lambda \psi^2} \left(\frac{K_{Ic}}{\sigma_u} \right)^2 \tag{6}$$

and of the generalized fracture toughness:

$$K_{Ic}^* = \lambda^\lambda \left[\frac{4\pi}{\psi^2} \right]^{(1-\lambda)} \frac{K_{Ic}^{2(1-\lambda)}}{\sigma_u^{1-2\lambda}} = \xi(\omega) \frac{K_{Ic}^{2(1-\lambda)}}{\sigma_u^{1-2\lambda}} \tag{7}$$

where the function ξ has been introduced for the sake of simplicity and the function ψ depends on the notch opening angle ω (its analytical expression can be found in Carpinteri *et al.*, 2007). The finite crack extension Δ and the generalized toughness K_{Ic}^* are both function of the material parameters and of the notch opening angle ω (through ψ and λ) and, hence, structural properties. Moreover, note that K_{Ic}^* is intermediate between strength and toughness (Leguillon, 2002), being the function ξ equal to 1 for ω equal to 0° and 180°.

Taking the ratio side by side of equations [4] and [5] and using the expression [7] of the generalized fracture toughness provided by the coupled FFM criterion, yields:

$$\frac{P_{cr}^{\omega}}{P_{cr}^{\pi}} = \xi(\omega) \frac{f\left(\frac{d}{b}, \pi\right)}{f\left(\frac{d}{b}, \omega\right)} s^{2(1-\lambda)} \tag{8}$$

The ratio between the failure loads for a notched and an un-notched specimen is therefore function only of the geometry and of the brittleness number s (equation [1]).

3.1. *Comparison with experimental data*

The coupled FFM criterion has been applied to the data obtained experimentally and presented in Section 2, in order to have an estimate of its effective predictive capability.

To what concerns *test 1*, wishing to have a more complete description of the effect of the notch opening angle, a FEA was performed by using the LUSAS ® code also for geometries that were not tested i.e. for ω equal to 0°, 30° and 90°. Results are presented in figure 3: as can be seen, the coupled FFM criterion provides satisfactory results and shows a minimum for notch opening angles larger than zero. This trend is in agreement with experimental data from the literature (see, for instance, Carpinteri, 1987; Seweryn, 1994).

In figure 4, theoretical predictions obtained by the coupled FFM criterion are plotted for different relative notch depths (*test 2*). Also in this case, a FEA was performed for geometries that were not tested: d/b=3/10 and 9/10. Results for intermediate geometries are in very good agreement with the experimental ones, the maximum percentage error keeping below 3%.

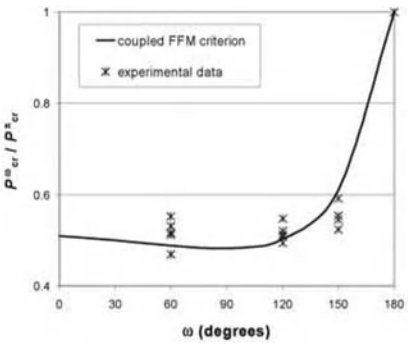

Figure 3. *Relative failure loads predictions of the TPB polystyrene specimens vs. notch opening angle*

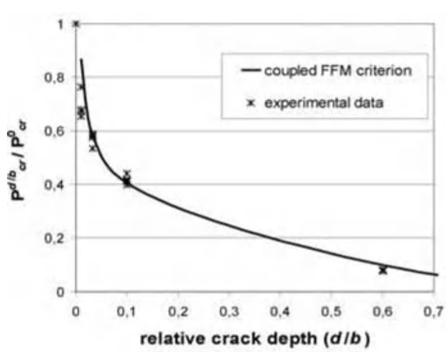

Figure 4. *Relative failure loads predictions of the TPB polystyrene specimens vs. relative crack depth*

On the other hand, for the extreme cases i.e. d/b=1/90 and 3/5, it rises up to 22%. Less good results for very small and very large notch depth were expected, since, in such cases, the finite crack extension tends to exceed the region of validity of the asymptotic stress field.

Eventually, test 3 shows a brittleness number s which is so high (i.e., a more ductile behaviour) that criteria based on the generalized SIF are no more applicable. Note that in Carpinteri *et al.* (2007) the coupled FFM criterion was successfully applied to the prediction of the failure load of duraluminum (less brittle than polystyrene) specimens. Anyway, because of the larger size of the specimens, in that case s was equal to 0.345, i.e. much lower than in the present case.

4. Conclusion

In this paper the results of a series of three-point bending tests on V-notched specimens made of polystyrene have been presented. Tests involved different notch angles, different notch depths and different dimensions of the samples. Hence, a large amount of experimental data concerning different geometries is now available. The coupled FFM criterion presented in Cornetti *et al.* (2006) is then taken into account. It has been stressed that this fracture criterion derives from an energy balance and is therefore more physically sound with respect to the classical average stress criterion. In order to check its validity, predictions by the coupled FFM criterion are compared with tests results: the agreement between theory and experiments is more than satisfactory.

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