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Generalizing the Random Walk and the Eddington–Weinberg laws for a hierarchical Universe

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Abstract

In this paper, we propose generalizations of the Random Walk and Eddington–Weinberg laws on the basis of a hierarchical, rather than fractal, geometry of the Universe. Evidence of the hierarchical evolution of the expanding Universe clearly emerges by a comparison with highly different-sized astrophysical, solar, biological and atomistic objects.

1. Introduction

The fractal Universe has a long story perhaps starting at the creation itself and in the literature with the work by the Swedish Astronomers Charliers, see [1]. Quantum Mechanics was probably the cause of primordial fluctuations in the energy–density, expanded up to the present era leading to the observed globular clusters, galaxies, clusters and superclusters of galaxies [2]. General relativity also plays a fundamental role; including scale relativity in the theory results in a continuous but non differentiable necessarily fractal space-time [3–5]. Thus, the real geometry of the Universe could be relativistic, quantum and fractal in nature [6–8].

We do not want here to compete with different and comparable viewpoints, such as the introduction in the Einstein–Hilbert gravitational action of scalar fields or curvature invariants [9–11]. We alternatively consider that the Universe has a memory of its quantum origin as suggested by Penrose [12]. Particularly, it is related to Penrose tiling and thus to the Cantorian space-time [13–15], as recently extensively discussed by Iovane [6–8] (see also related references).

Perhaps the two most important scaling laws in this context are probably: (i) the well-known Random Walk or Brownian Motion equation, firstly used by Eddington, recently validated also in the context of astrophysical objects [16] (on the basis of the time-statistical fluctuations appearing as a consequence of the chaotic dynamics in multi-particle systems) even if deviations are observed [6], and: (ii) the Eddington–Weinberg correlation among the fundamental constants, even if its actual formulation cannot be straightforwardly (i.e. substituting the actual radius of the Universe with its generic size) extended. In addition, fundamental scaling laws are suggested in different fields such as mechanics [17] or biology [18], as we have recently discussed [19] (see also related references).

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2. The generalized Random Walk law in an expanding hierarchical Universe

In this paper, we propose a generalization of these scaling laws, assuming a hierarchical (not self-similar), rather than fractal, geometry of the Universe. The hierarchical geometry of the Universe is depicted in Fig. 1, in which clusters, clusters of clusters (super clusters) and so on, can be recognized. The smallest units, at the level 0, are considered scale-invariant, e.g. nucleons of mass m (the mass difference between protons and neutrons and the mass of electrons are here neglected). Each cluster at the level $k+1$ contains n_k sub-clusters, each of them with volume V_k. Note that $\varphi_k = \frac{n_{k-1}V_{k-1}}{V_k}$ represents the volumetric fraction of sub-clusters at the level $k-1$ in a cluster at the level k. Thus, the total number of clusters at the level k is $N_k = \prod_{j=1}^{k} n_j$, with a volumetric fraction $\phi_k = \prod_{j=1}^{k} \varphi_j$. The hierarchical mass balance, up to the final level L, implies $M \equiv M_L = N_L \rho_L V_L = N_k\, \rho_k V_k, \forall k$, where ρ_k is the density observed at the level k. Let us search a hierarchical solution as a perturbation of a fractal solution, for which $n_k = n$ and $\varphi_k = \varphi$, thus k-independent numbers and fractions; accordingly $N_k = n^k$ and the total number of nucleons is $N \equiv N_L = n^L$. If the clusters present a fractal distribution, we expect $M(r) \propto r^D$, where $r \equiv r_L = (V_L)^{(1/3)}$ is the characteristic size of the considered zone and D is the constant (an hypothesis that we are going to verify and then relax) fractal dimension. Note that the mass M or the energy Mc^2 (c is the speed of light) has a scaling identical to that of the energy dissipated in solids during defect nucleation (cracks or dislocations; in this analogy galaxies are thus defects of the space-time) or to the metabolic energy spent by living organisms [19]. The constant of proportionality can be deduced noting that $M = Nm$, thus $r \propto N^{1/D}$, and $r(N=1) \approx \lambda_C$, that is the Compton wavelength ($\lambda_C = h/(mc)$, where h is the Planck's constant); accordingly:

$$r \approx \lambda_C N^{1/D}. \tag{1}$$

Eq. (1) is an extension of the well-known Random Walk or Brownian Motion equation, identically recovered for $D = 2$. The validity of Eq. (1) with the exponent 1/2 has recently been demonstrated in the astrophysical context [16], whereas a variability of the exponent has been observed in [6], towards 1/3; such a value, quite intuitive since it corresponds to a constant density, has been also derived from first physical principles for planets, white dwarfs and neutron stars [20]. Note that Eq. (1) interpreted in the light of the Fibonacci number or Golden Mean [6] would suggest $D = 2/(\sqrt{5}-1) \approx 1.6$.

Writing $N = (r/\lambda_C)^D = n^L$ we derive $L = D\ \ln(r/\lambda_C)/\ln n$, that defines the number of hierarchical levels. Such a law shows that only few hierarchical levels are required for spanning several orders of magnitude in size. For example, for a fractal Universe, considering for $r = R \approx 10^{26}$ m its actual radius, and $\lambda_C \approx 10^{-15}$ m, $n \approx 10^4$ (number of galaxies in a cluster) and $D = 2$ would result in only ~20 hierarchical levels.

The fractal exponent D can be determined noting that $\phi \equiv \phi_L = \varphi^L = NV_0/V$ represents the macroscopic volumetric fraction of nucleons. Thus, we derive $r/\lambda_C = (n/\varphi)^{L/3}$. Introducing this result into the expression for L provides the fractal exponent, as a function of well-defined physical quantities:

$$D = \frac{3\ln n}{\ln n - \ln \varphi}. \tag{2a}$$

Since $n \geqslant 1$ and $0 \leqslant \varphi \leqslant 1$ we expect $0 \leqslant D \leqslant 3$. A constant value of D represents a fractal Universe. A hierarchical Universe would have a slightly variable value of D, as a consequence of a variable value of n and/or of φ. Note that $\phi = \rho_L/\rho_C$, where $\rho \equiv \rho_L$ is the density (e.g., if $r = R$, of the actual Universe) and ρ_C is the density of a nucleon; accordingly:

$$\varphi = (\rho/\rho_C)^{1/L}. \tag{2b}$$

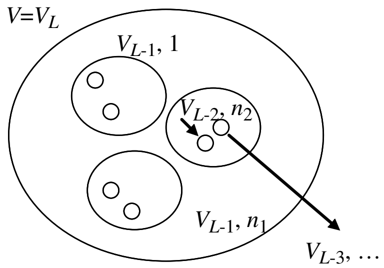

Fig. 1. Scheme of a hierarchical Universe.

Thus, fixing the number L of hierarchical levels (an hypothesis that can be easily relaxed) and considering and expanding Universe (ρ/ρ_C from one tends to zero) the model naturally predicts a transition from $D = 3$ to $D = 0$, thus a hierarchical rather than a fractal Universe, as a consequence of its expansion.

Following [6] and to confirm this predicted weak decay of the fractal dimension by increasing the size-scale, we have deduced D by fitting Eq. (1) to the observed mass–radius relations for astrophysical clusters, solar system objects, organic matter and single elements (Table 1). A relevant agreement and a clear transition of the fractal dimension from $\sim$2.8 in atoms to a value of $\sim$1.9 in superclusters of galaxies is observed.

From Eq. (1) we deduce $M/M_{L-1} = (r/r_{L-1})^D$: this, self-consistently, suggests a positive value of D; moreover, considering $L-1$ as describing a supercluster of galaxies (see Table 1; $r = R \approx 10^{26}$ m) we can evaluate the number of superclusters in our Universe: considering $D \approx 1.9$ (as observed in superclusters) results in 6310–501187 superclusters, corresponding to an actual total mass of $\sim 10^{49}$–10^{53} kg or, for $D \approx 1.6$ (Fibonacci number/Golden Mean) 1585–63096 superclusters, corresponding to a actual total mass of $\sim 10^{48}$–10^{52} kg.

3. The generalized Eddington–Weinberg law in an expanding hierarchical Universe

According to the analysis reported in [16] the minimal unit of action is $h \approx A/N^{3/2}$, where A is the total action of a system composed by N particles, in contrast to its mean unit value $a = A/N$. Thus, $a \approx h\sqrt{N}$ was considered in [16] to derive Eq. (1) with $D = 2$, i.e. the Random Walk equation. Accordingly, we expect in general $h \approx A/N^{(D+1)/D}$. The Virial Theorem states $2T + U = 0$ (between two quasi-equilibrium configurations), where $U = -GM(R)m/R$ is the gravitational potential (G is the gravitational constant), $T = 1/2mv^2$ is the kinetic energy and $v = HR$ is the velocity, where H is the Hubble's constant. Thus $H = \sqrt{G\rho}$, from which, noting that $A \approx -U/H$, we find:

$$h \approx \sqrt{GRm^3 N^{3/2-(D+1)/D}}. \tag{3}$$

Table 1
Evidence of a hierarchical Universe: variable fractal exponent by varying the size-scale

Object	r (m)	M (kg)	D
H (i.e., 1 nucleon)	1.50×10^{-15}	1.67×10^{-27}	–
He	2.38×10^{-15}	6.64×10^{-27}	2.342
Li	2.86×10^{-15}	11.52×10^{-27}	2.498
Be	3.12×10^{-15}	14.96×10^{-27}	2.549
B	3.32×10^{-15}	17.94×10^{-27}	2.574
C	3.43×10^{-15}	19.94×10^{-27}	2.597
Cr	5.60×10^{-15}	86.31×10^{-27}	2.730
Mn	5.70×10^{-15}	91.20×10^{-27}	2.735
Fe	5.73×10^{-15}	92.71×10^{-27}	2.736
Lr	9.57×10^{-15}	431.60×10^{-27}	2.804
Prokaryotic cell	$\sim 10^{-6}$–10^{-5}	$\sim 10^{-9}$–10^{-8}	1.799–2.002
Eukaryotic cell	$\sim 10^{-5}$–10^{-4}	~ 1–4×10^{-7}	1.818–2.002
Man	$\sim$1, 0.3	$\sim 10^2$, 80	1.934, 1998
Pluto	1.14×10^6	1.79×10^{22}	2.351
Moon	1.74×10^6	7.35×10^{22}	2.365
Mercury	2.44×10^6	3.29×10^{23}	2.363
Mars	3.39×10^6	6.39×10^{23}	2.376
Venus	6.05×10^6	4.87×10^{24}	2.377
Earth	6.38×10^6	5.98×10^{24}	2.365
Uranus	25.60×10^6	8.67×10^{25}	2.370
Neptune	24.75×10^6	1.03×10^{26}	2.363
Saturn	59.65×10^6	5.69×10^{26}	2.378
Jupiter	71.40×10^6	1.90×10^{27}	2.406
Sun	6.96×10^8	1.99×10^{30}	2.342
Globular clusters	$\sim 10^{17}$	$\sim 10^{36}$–10^{37}	2.351
Galaxies	$\sim 10^{19}$–10^{20}	$\sim 10^{40}$–10^{42}	1.915–1.971
Cluster of galaxies	$\sim 10^{22}$	$\sim 10^{45}$	1.946
Supercluster of galaxies	$\sim 10^{23}$–10^{24}	$\sim 10^{45}$–10^{47}	1.846–1.895
Universe	$\sim 10^{26}$	$\sim 10^{48}$–10^{53} (estimated)	1.6–1.9 (assumed)

Eq. (3) is an extension of the well-known Eddington–Weinberg law, identically recovered for $D = 2$, which suggests a scaling law for the gravitational "constant" G. Obviously, one could mix Eqs. (1), (2) and (3).

4. Conclusions

Summarizing, in this paper we have proposed generalizations of the classical Random Walk and Eddington–Weinberg laws ("incidentally" valid only in our era, for which $D \approx 2$), Eqs. (1–3), for a hierarchical Universe (Fig. 1). According to the analysis just after the creation ($D \approx 3$) the Random Walk and Eddington–Weinberg laws were not valid and to be substituted with $r \approx \lambda_C N^{1/3}$ and $h \approx \sqrt{GRm^3 N^{1/6}}$ (where G could differ with respect to its current value), whereas in the future we expect a transition towards an era dominated by the vacuum ($D = 0$, even if higher asymptotic values for D are possible, e.g. the Golden Mean). Evidence of this hierarchical nature and evolution of our Universe emerges by a comparison with highly different-sized astrophysical, solar, biological and atomistic objects (Table 1). The proposed generalizations suggest new scaling laws on the gravitational and Hubble's "constants" (work in progress).

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