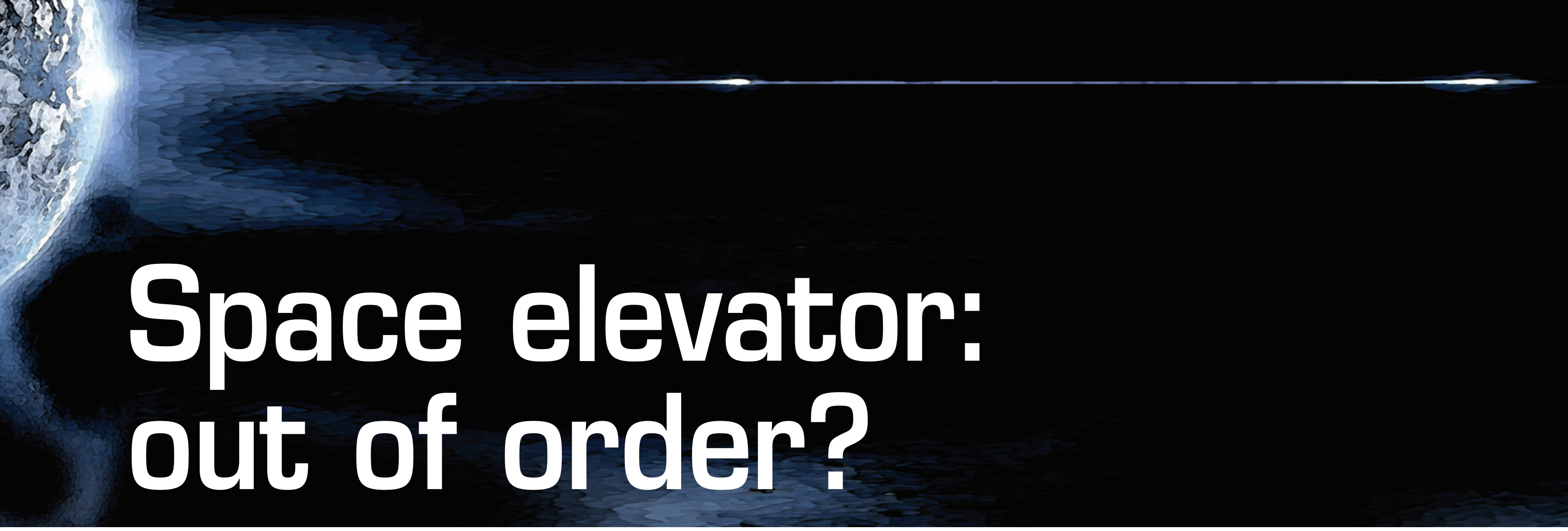


Space elevator: out of order?

Classical theories of the strength of solids, such as fracture mechanics or those based on the maximum stress, assume a continuum. Even if such a continuum hypothesis can be shown to work at the nanoscale for elastic calculations, it has to be revised for computing the strength of nanostructures or nanostructured materials. Accordingly, quantized strength theories have recently been developed and validated by atomistic and quantum-mechanical calculations or nanotensile tests. As an example, the implications for the predicted strength, today erroneously formulated, of a carbon-nanotube-based space elevator megacable are discussed. In particular, the first *ab initio* statistical prediction for megacable strength is derived here. Our findings suggest that a megacable would have a strength lower than ~45 GPa.

Nicola M. Pugno

Department of Structural Engineering and Geotechnics, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy.
E-mail: nicola.pugno@polito.it

A space elevator[1] consists of a cable attached to a planet surface for carrying payloads into space. If the cable is long enough, centrifugal forces exceed gravitational forces and the cable will work under tension; for the Earth this critical length[2] is of the order of 150 Mm.

Fig. 1 shows artistic representations of the space elevator cable and concept. The cable would be anchored to a base station that could be mobile or fixed. Mobile platforms could be controlled to avoid high winds, storms, and space debris, but fixed base stations would be cheaper. For planar ribbon cables, climbers (with pairs of rollers to hold the cable with friction or smart gecko robots[3]) would carry the payloads into space. Laser power beaming could also be adopted to sustain and control the energy required by the climber. However, the climber itself would be naturally accelerated after reaching the geosynchronous orbit and could thus accumulate energy or exchange it with a different climber. The elevator would stay fixed geosynchronously. Thus, a space elevator could revolutionize the carrying of payloads into space, but its design is very challenging.

The most critical component in the space elevator design is undoubtedly the megacable, which requires a material with very high strength-density ratio[2]. Carbon nanotubes (CNTs)[4] are ideal candidates to build such a megacable[5,6] because of their low density and huge strength, recently measured by nanotensile tests[7,8]. But basing the design of the space-elevator cable on the theoretical strength of a single CNT[5,6] is naive[9–11]. Accordingly, we present here the first *ab initio* derivation of the statistical prediction for

ISSN:1748 0132 © Elsevier Ltd 2007

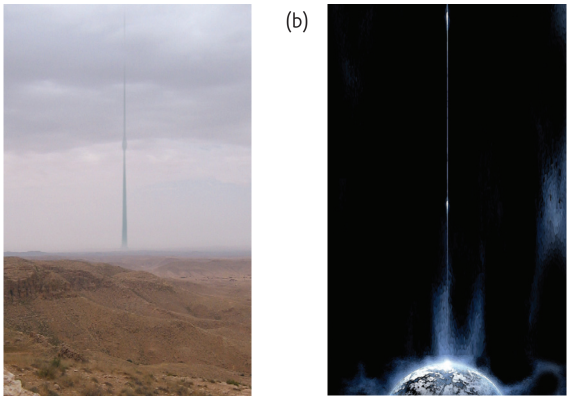
(a) 

Fig. 1 The space elevator concept. Artistic representations (a) from the Earth and (b) from space. (Courtesy of StudioAta, Torino, Italy.)

megacable strength, and a corresponding flaw-tolerant design is proposed.

For a cable with constant cross section and a vanishing tension at the planet surface, the maximum stress-density ratio for the Earth (reached at the geosynchronous orbit) is 63 GPa/(1300 kg/m^3). This corresponds to 63 GPa, if the low density of carbon is assumed for the cable. Such a large failure stress has been measured experimentally[7,8] during tensile tests of ropes composed of single- or multiwalled CNTs, both expected to have an ideal strength of ~100 GPa. Note that for steel (density 7900 kg/m^3, maximum strength 5 GPa), the maximum stress expected in such a cable would be 383 GPa, whereas for kevlar (density 1440 kg/m^3, strength 3.6 GPa) it would be 70 GPa. Both values are much higher than their strengths[9].

However, an optimized cable design would have a uniform tensile stress profile rather than a constant cross-sectional area[2]. Accordingly, the cable could be built of any material by using a large enough taper ratio, i.e. the ratio between maximum (at geosynchronous orbit) and minimum (at the Earth's surface) cross-sectional areas. A giant and unrealistic taper ratio would be required for steel or kevlar (10^{33} or 2.6×10^8, respectively), whereas for CNTs, it is theoretically[6] only 1.9. Thus, the feasibility of the space elevator seems to become only plausible[5,6] thanks to the discovery of CNTs. The cable would represent the largest engineering structure ever created, ~20 times longer than the Great Wall of China, with a hierarchical design from the nano- (single CNT) to the megascale (cable of a hundred megameters).

Strength of nanotubes or bundles

Local theories have to be rejected in order to compute the strength of a structure properly, since they are unable to predict size effects because of the lack of a characteristic internal length (for details, see supplementary material and references[12–15]). For example, computing the tensile failure of a linear elastic infinite plate containing a hole using a maximum-stress local approach will always give one third of the defect-free strength, because a local theory cannot distinguish between a 'small' or 'large' (with respect to what?) hole. In contrast, quantized fracture mechanics (QFM)[16–18] is derived from classical linear elastic fracture mechanics (LEFM)[19] by removing the hypothesis of the continuous crack growth and thus naturally introducing an internal characteristic length, namely the fracture quantum. Discrete crack advancement is a material/structural property and is expected to increase with the size scale[16–18]. However, atomistic simulations reveal that the fracture quantum in CNTs is close to the distance between two broken adjacent chemical bonds[16–18]. QFM can treat different defect sizes and shapes in a simple analytical way and not only the 'long' sharp cracks of LEFM.

Using QFM, the ratio between the failure stress, σ_N, of a defective nanotube and its defect-free strength, $\sigma_N^{(theo)}$, (i.e. theoretical strength as computed, for example, by stretching pristine nanotubes using *ab initio* simulations based on density functional theory) can be calculated. This is done by equating the mean value along the fracture quantum of the energy release rate with the fracture energy per unit area of carbon. For a nanotube having a fracture quantum q (the 'atomic size') and containing an elliptical hole of half-axes a and b (a is perpendicular to the applied load or nanotube axis), we find (Fig. 2):

$$\frac{\sigma_N(a,b)}{\sigma_N^{(theo)}} = \sqrt{\frac{1+2a/q\,(1+2a/b)^{-2}}{1+2a/q}} \qquad (1)$$

For transverse cracks having length m (in fracture quanta units), $\sigma_N(m)/\sigma_N^{(theo)} \approx (1+m)^{-1/2}$ ($b \approx 0$, $m \approx 2a/q$). We have neglected tip-tip and tip-boundary interactions here, which would further reduce the failure stress. Better predictions could be derived by considering the energy release rate[20] at the tip of a crack in a finite-radius cylinder.

Imposing the force equilibrium for a cable composed of defective nanotubes allows derivation of the cable strength σ_C (where the defect-free strength is $\sigma_C^{(theo)}$):

$$\frac{\sigma_C}{\sigma_C^{(theo)}} = \sum_{a,b} f_{ab}\,\frac{\sigma_N(a,b)}{\sigma_N^{(theo)}} \qquad (2)$$

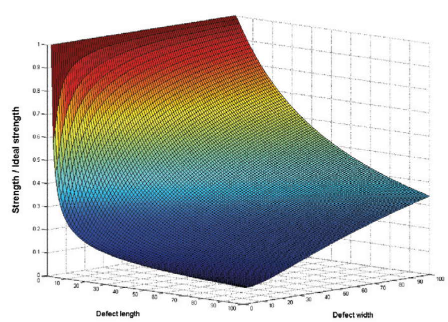


Fig. 2 Ratio between failure stress and ideal strength ($\sigma_N/\sigma_N^{(theo)}$) as a function of length and width of the elliptical defect in units of fracture quanta, q. The curve obtained for vanishing defect width and large defect length corresponds to the classical Griffith's theory of fracture, while predictions for large defect length and width are identical to the inverse of the classical stress concentrations for elasticity. Nanotensile tests on nanotubes give $\sigma_N/\sigma_N^{(theo)} \approx 0.1$–$0.7$, thus the related horizontal planes identify plausible solutions for defect sizes and shapes (Table 1, supplementary material).

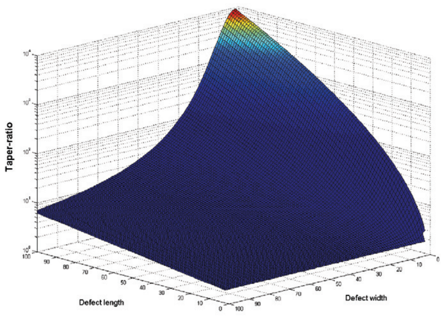


Fig. 3 Taper ratio, λ, as a function of length and width of the elliptical defect in units of fracture quanta, q. Taper ratios several orders of magnitude larger than the theoretical value would have to be used in a flaw-tolerant cable design.

The summation is extended to all holes, where f_{ab} is the numerical fraction of CNTs containing an elliptical hole of half-axes a and b (f_{00} is the numerical fraction of defect-free nanotubes and $\sum_{a,b} f_{ab} = 1$). If all the defective nanotubes in the bundle contain identical holes, $f_{ab} = f = 1 - f_{00}$, and the following simple relation holds: $1 - \sigma_C/\sigma_C^{(theo)} = f(1 - \sigma_N/\sigma_N^{(theo)})$.

Thus, a taper ratio[2], λ, larger than its theoretical value would be required for a megacable to be flaw tolerant[10] against the propagation of an elliptical hole. The flaw-tolerant taper ratio is (Fig. 3):

$$\lambda = \lambda^{(theo)\,\frac{\sigma_C^{(theo)}}{\sigma_C}} \qquad (3)$$

Thus, designing a megacable with the theoretical taper ratio as currently proposed would surely lead to its failure.

By applying our treatment to results of nanotensile tests on CNTs[7,8,21], we can identify plausible sizes and shapes for the most critical defect causing nanotube fracture (Fig. 2 and Table 1, supplementary material). Assuming these plausible defects occur in the CNTs in the megacable, we deduce flaw-tolerant taper ratios orders of magnitude larger than the theoretical value (Fig. 3). Defects are expected in the bundle, simply for thermodynamic[10] and statistical[22] reasons, but also as a result of space debris impacts[9] and other forms of damage accumulation.

Fatigue of nanotubes or bundles

The space elevator cable would be cyclically loaded, e.g. by climbers carrying payloads, thus fatigue could play a role in its design. By integrating the quantized Paris law[18,23,24] (an extension of the classical Paris law[25] that has recently been proposed especially for nanostructure or nanomaterial applications), we derive the following number of cycles $C_N(a)$ to failure (where the defect-free number of cycles is $C_N^{(theo)}$), assuming a pre-existing crack of half-length a:

$$\frac{C_N(a)}{C_N^{(theo)}} = \frac{(1+q/W)^{1-p/2} - (a/W + q/W)^{1-p/2}}{(1+q/W)^{1-p/2} - (q/W)^{1-p/2}}, \quad p \neq 2 \qquad (4)$$

$$\frac{C_N(a)}{C_N^{(theo)}} = \frac{\ln\{(1+q/W)/(a/W + q/W)\}}{\ln\{(1+q/W)/(q/W)\}}, \quad p = 2 \qquad (5)$$

where $p > 0$ is the Paris exponent of the material and W is the strip width. Note that according to Wöhler[26] $C_N^{(theo)} = K\Delta\sigma^{-k}$, where K and k are material constants and $\Delta\sigma$ is the amplitude of the stress range during oscillations. Even though fatigue experiments in nanotubes are still to be performed, their behavior is expected to be intermediate between those of Wöhler and Paris (as displayed by all known materials). The quantized Paris law basically represents their asymptotic matching (as QFM basically represents the asymptotic matching between strength and energy/toughness approaches).

Only defects remaining self-similar during fatigue growth have to be considered, thus only a crack (of half-length a) is of interest in this context. Using eqs 4 and 5, the time to failure of a nanotube can be estimated, in a similar way to the brittle fracture of eq 1.

For a CNT bundle, a mean-field approach (similar to eq 2) yields the number of cycles to failure C_C (where the defect-free number of cycles is $C_C^{(theo)}$) of a cable containing nanotubes with pre-existing cracks of half-length a in vacancy fractions f_a:

$$\frac{C_C}{C_C^{(theo)}} = \sum_a f_a \frac{C_N(a)}{C_N^{(theo)}} \qquad (6)$$

Eq 6 allows us to calculate the life-time reduction $(1 - C_C/C_C^{(theo)})$ of a CNT bundle by the presence of a given crack-size distribution in the nanotubes. Better predictions could be derived by integrating the quantized Paris law for a finite-width strip. However, we note that the role of finite width is already included in eqs 4–6, even if these are rigorously valid in the limit of W tending to infinity.

Superplasticity or hyperelasticity

The equations above are based on linear elasticity, i.e. a linear relationship between stress, σ, and strain, ε, where $\sigma \propto \varepsilon$. In contrast, a nonlinear constitutive law, $\sigma \propto \varepsilon^{\kappa}$ ($\kappa \neq 1$), is more appropriate to treat the superplasticity ($\kappa \to 0^+$) or elastic-plasticity ($0 < \kappa < 1$) recently observed in CNTs[21], as well as hyperelastic materials ($\kappa > 1$).
The power to which the stress-singularity at the tip of a crack is raised in such a nonlinear material would then be modified[27] from the classical value of ½ to $\alpha = \kappa/(\kappa + 1)$. That is, the asymptotic stress field at the crack tip scales with the distance r from the tip as $r^{-\alpha}$, with $0 \leq \alpha \leq 1$. Thus, the problem is mathematically equivalent to that of a edge corner defect[28], and consequently we predict:

$$\frac{\sigma_N(a,b,\alpha)}{\sigma_N^{(theo)}} = \left(\frac{\sigma_N(a,b)}{\sigma_N^{(theo)}}\right)^{2\alpha}, \quad \alpha = \frac{\kappa}{\kappa + 1} \qquad (7)$$

Plasticity reduces the severity of the defect, vanishing for superplastic materials, whereas hyperelasticity increases its effect. For a crack composed of m adjacent vacancies, for example, we find: $\sigma_N/\sigma_N^{(theo)} \approx (1 + m)^{-\alpha}$. Eq 7 can be used to correct predictions for the fracture strength of nonlinear materials given by eqs 1 and 2 (and eq 3), or fatigue strength by substituting $p/2$ with p/α in eqs 4–6.

Maximum achievable strength

Defects are thermodynamically unavoidable, especially at the megascale[10]. At thermal equilibrium, the vacancy fraction, $f = n/N$, where n is the number of vacancies and N is the total number of atoms, is estimated to be:

$$f \approx e^{-E_1/k_B T_a} \quad (8)$$

where $E_1 \approx 7$ eV is the energy to remove one carbon atom and T_a is the absolute temperature at which the carbon is assembled (typically 2000–4000 K). Thus, f is in the range ~2.4×10^{-18}–1.6×10^{-9}. For the megacable with a carbon weight of ~5000 kg, the total number of atoms is $N \approx 2.5 \times 10^{29}$. Thus a huge number of equilibrium defects in the range $n \approx 0.6 \times 10^{12}$–$3.9 \times 10^{20}$ are expected, in agreement with recent discussions[29] and observations[30].

The strength of a cable will be dictated by the largest transverse crack in it, according to the weakest link concept. The probability of finding a nanocrack of size m in a bundle with vacancy fraction f is $P(m) = (1 - f)f^m$. Thus the number M of such nanocracks in a bundle composed of N atoms is $M(m) = P(m)N$. The size of the largest nanocrack, which typically occurs once, is found from the solution of the equation $M(m) \approx 1$, which implies[31]:

$$m \approx -\ln[(1-f)N]/\ln f \approx -\ln N/\ln f \quad (9)$$

Thus we deduce a size $m \approx 2$–4 for the largest thermodynamically unavoidable defect in a megacable. Inserting eqs 8 and 9 into eq 1 for a transverse crack ($b \approx 0$ and $m \approx 2a/q$), we deduce the statistical counterpart of eq 1 and the following thermodynamically maximum achievable strength:

$$\frac{\sigma_N(N)}{\sigma_N^{(theo)}} \leq \frac{\sigma_N^{(max)}(N)}{\sigma_N^{(theo)}} = \frac{1}{\sqrt{1+\frac{k_B T_a}{E_1}\ln N}} \quad (10)$$

Inserting eq 10 into eqs 2 and 3, the maximum cable strength and minimum taper ratio can be deduced statistically. The maximum achievable strength, an unavoidable limit at least at thermodynamic equilibrium, is ~45 GPa and the flaw-tolerant taper ratio is ~4.6. But the larger taper ratio implies a large cable mass and thus a large number N of atoms. Updating N in our statistical calculation yields the same, thus self-consistent, predictions. Statistically[32], we expect an even smaller strength[9,10].

Conclusions

We have presented key formulas for the design of a flaw-tolerant space elevator megacable, suggesting that it would have a lower strength, or would need a larger taper ratio, than has been previously proposed. The thermodynamic maximum achievable strength, which does not involve any best-fit parameter, has been derived to calculate the first *ab initio* statistical prediction of the megacable strength. This is expected to be <45 GPa. A strength of ~10 GPa (which has been experimentally observed in individual CNTs), for example, would dramatically increase the taper ratio to ~613. Thus, is the space elevator out of order? Our opinion is: at present, yes; but never say never. However, our proposed flaw-tolerant concept could be key for a terrestrial space elevator design far in the future. Moreover, a lunar space elevator, because of the lower gravity, could perhaps be realized with existing engineering materials and an opportune flaw-tolerant structural design. Tethered space systems, pioneered by Grossi and Colombo in 1972, are becoming even more intriguing in the new era of nanomaterials. nt

REFERENCES

1. Artsutanov, Y. V., Kosmos na Elektrovoze, *Komsomol-skaya Pravda*, July 31, 1960; described in Lvov, V., *Science* (1967) **158**, 946
2. Pearson, J., *Acta Astronautica* (1975) **2**, 785
3. Pugno, N., *J. Physics: Condens. Matter* (2007) **19**, 395001
4. Iijima, S., *Nature* (1991) **354**, 56
5. Edwards, B. C., *Acta Astronautica* (2000) **10**, 735
6. Edwards, B. C., and Westling, E. A., *The Space Elevator: A Revolutionary Earth-to-Space Transportation System*, Spageo Inc., (2003)
7. Yu, M. F., *et al.*, *Phys. Rev. Lett.* (2000) **84**, 5552
8. Yu, M. F., *et al.*, *Science* (2000) **287**, 637
9. Pugno, N., *J. Physics: Condens. Matter* (2006) **18**, S1971
10. Pugno, N., *Acta Mater.* (2007) **55**, 5269
11. Palmer, J., The space elevator: going down?, *Nature News*, May 22 2006, doi:10.1038/news060522-1
12. Novozhilov, V. V., *Prik. Mat. Mek.* (1968) **33**, 212
13. Ippolito, M., *et al.*, *Phys. Rev. B* (2006) **73**, 104111
14. Ippolito, M., *et al.*, *Phys. Rev. B* (2007) **75**, 224110
15. Taylor, D., *et al.*, *Eng. Fracture Mech.* (2005) **72**, 1021
16. Pugno, N., and Ruoff, R., *Philos. Mag.* (2004) **84**, 2829
17. Pugno, N., *Int. J. Fracture* (2006) **140**, 159
18. Pugno, N., *Int. J. Fracture* (2006) **141**, 311
19. Griffith, A. A., *Philos. Trans. R. Soc. London, Ser. A* (1920) **221**, 163
20. Murakami, H., *Stress Intensity Factors Handbook*, Pergamon, Oxford, (1986)
21. Ding, W., *et al.*, *Exp. Mech.* (2006) **47**, 25
22. Carpinteri, A., and Pugno, N., *Nat. Mater.* (2005) **4**, 421
23. Pugno, N., *et al.*, *J. Mech. Phys. Solids* (2006) **54**, 1333
24. Pugno, N., *et al.*, *Eng. Fracture Mech.* (2007) **74**, 595
25. Paris, P., and Erdogan, F., *Trans. ASME J Basic Eng.* (1963), 528
26. Wöhler, A., *Z. Bauwesen* (1860), 10
27. Rice, J. R., and Rosengren, G. F., *J. Mech. Phys. Solids* (1968) **16**, 1
28. Carpinteri, A., and Pugno, N., *Eng. Fracture Mech.* (2005) **72**, 1254
29. Bhadeshia, H. K. D. H., *Mater. Sci. Technol.* (2005) **21**, 1293
30. Fan, Y., *et al.*, *Nat. Mater.* (2005) **4**, 906
31. Beale, P. D., and Srolovitz, D. J., *Phys. Rev. B* (1988) **37**, 5500
32. Carpinteri, A., *Int. J. Solids Struct.* (1994) **31**, 291