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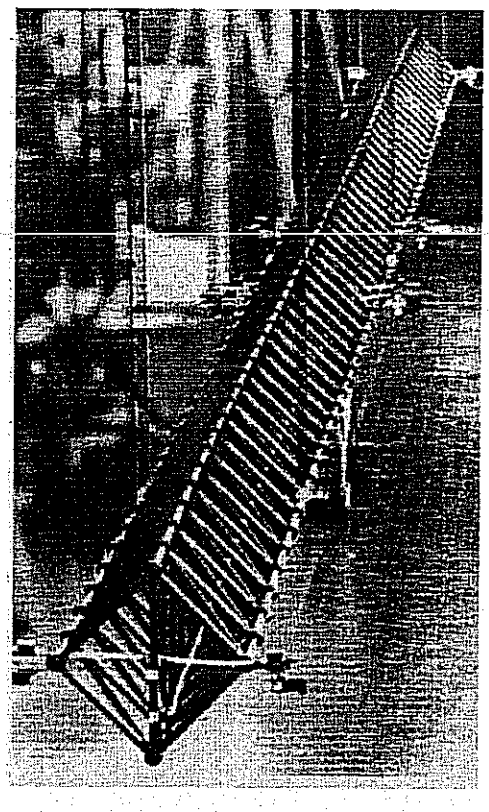

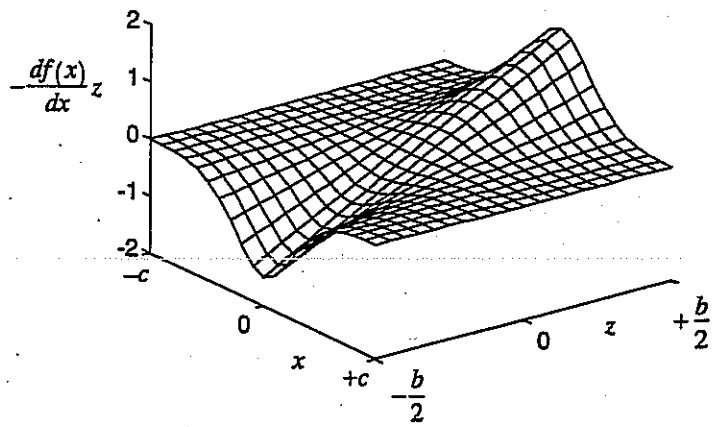

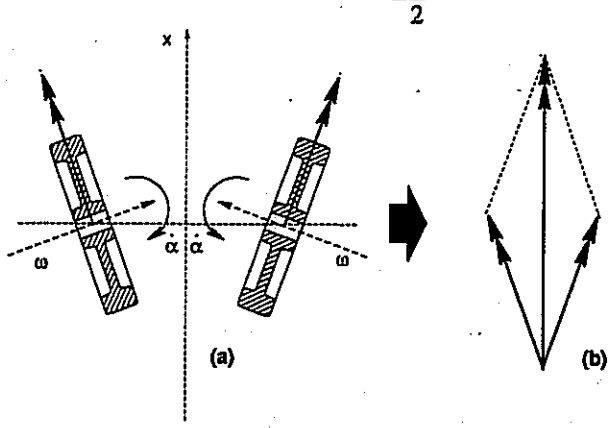

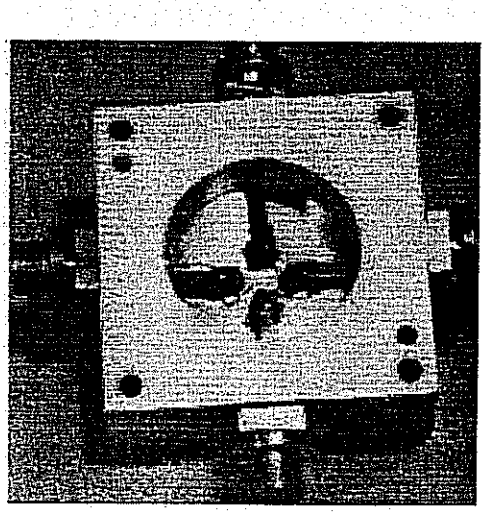

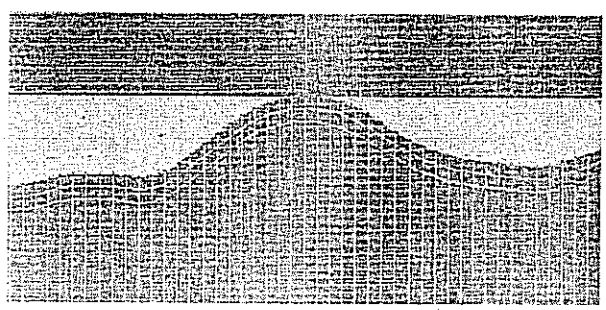

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Closed form solution for non tubular bonded joints with tapered adherends under torsion

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Keywords: **bonded joint, non-tubular, tapered adherends, torsion.**

Abstract

The object of the present paper is the study of an adhesive bonded non tubular joint with tapered adherends subjected to torsion. The joint considered consists of two rectangular section beams bonded together along a side surface.

The work emphasized how the stress field developed in the adhesive at the bond area between the two beams is heavily dependent on joint geometry: the maximum stresses for a joint without tapered adherends are reached at the ends of the adhesive while they appear along its centerline if the adherends are tapered.

The analytical equations which govern torsional moment section by section in the joint, stresses and strains in the adhesive and the joint's elastic strain (rotation between the two beams) are obtained as power series with coefficients given in closed recursive form.

1 Introduction

As the literature indicates, studies on adhesive bonded joints have hitherto concentrated on the effects of tension or compression, flexure and shear. In certain situations, however, such joints are also subject to torsion. Research in this area is restricted to tubular structures, and only recently non-tubular bonded joints have been studied. [15-18].

Since the two pioneer papers of Goland and Reissner [7] and Lubkin and Reissner [12] tubular joints subject to torsion have been studied from many different points of view.

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Theoretical approaches have been validated experimentally and numerically; Fracture Mechanics has been used to solve the problem of the joint's strength in the case of brittle collapse with static or fatigue loading; the non-linear and viscoelastic adhesive's behaviour has been considered and recently also the dynamic analysis have been carried out. Specifically in the works of Hipol [9] and Chen and Cheng [5] it is emphasized the positive influence of tapering at the adherends' end on the stress peaks in the adhesive layer respectively from a numerical and a theoretical point of view: if the adherends are partially-tapered the stress peaks become lower.

The lack of work on non-tubular joints indicated by this bibliography motivated the investigation presented in the work developed by Pugno [16] and Pugno, Surace [17]. Pugno and Surace [17] have studied a non tubular joint subjected to torsion from a theoretical point of view validated numerically by a three dimensional finite element analysis.

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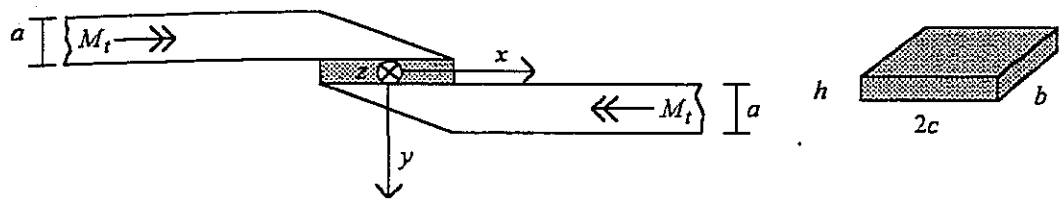

Fig. 1 Rectangular tapered adherends in a non-tubular bonded joint under torsion

In Pugno's paper [16] the non tubular joint is optimized for uniform torsional strength: starting from a non tapered joint the optimization was achieved by chamfering the edges, which are in any case not involved in the stress flow induced by the tensile loading for which the joint should be designed. The resulting optimized joint is thus both lighter and stronger. The same results for tubular joints is presented in the work of Pugno and Surace [18], where the joint is optimized for uniform torsional strength.

The present paper wants to extend to the study of a non tubular joint subjected to torsion in the case of tapered adherends.

2 Mathematical model

It is assumed that the materials making up the joint (Figure 1) are governed by a linear elastic law (in particular the materials are assumed isotropic). While this is intuitively obvious for the beams (which are typically metal), this is not the case for the adhesive, which is more likely to show a typically non-linear elastic behaviour.

Under torsion, however, the stress state in the adhesive relative to a non-tubular bonded joint is basically normal. As it is well known that adhesive can withstand tangential stresses which are an order of magnitude higher than the ultimate normal stresses, it follows that the normal stresses occurring in the adhesive during service must be limited. It is precisely because of these modest stresses that we can assume that the adhesive is also governed by a linear elastic law and that it is verify also the geometrical linearity. The adherends can be studied with the Technical Theory of Beams.

Assuming linear elastic laws and using equilibrium and compatibility equations to solve the torsional problem [16,17], the function which governs transmission of the torsional moment along the overlap, or in other words, how the torsional moment progresses section by section along the top beam, can be found by solving the following differential equation:

$$\frac{d^2M(x)}{dx^2}+\frac{K^*}{G}\frac{I_{t_1}(x)+I_{t_2}(x)}{I_{t_1}(x)I_{t_2}(x)}M(x)= -\frac{K^*M_t}{GI_{t_2}(x)}\begin{cases}M(x=-c)=M_t\\ M(x=c)=0\end{cases}, \quad (1)$$

where x is the longitudinal coordinate of the joint (Figure 1).

The torsional moment on the top beam found section by section in the thin adhesive layer is designated as M, while K^* is the adhesive specific (per unit length) stiffness, G is the shear elastic modulus of the two beams and I_t is their factor of torsional rigidity.

The adhesive specific stiffness can be expressed as:

$$K^*=\frac{(1-\nu_a)E_a}{(1+\nu_a)(1-2\nu_a)}\frac{b^3/12}{h}=E_a^*\frac{I_x^*}{h}, \quad (2)$$

where ν_a, E_a are the Poisson's ratio and Young's modulus respectively of the adhesive, h is its thickness, b is the base of the beam's cross section (Figure 1), while $I_x^*=b^3/12, E_a^*$ indicate respectively a specific moment of inertia and the adhesive's fictitious Young modulus.

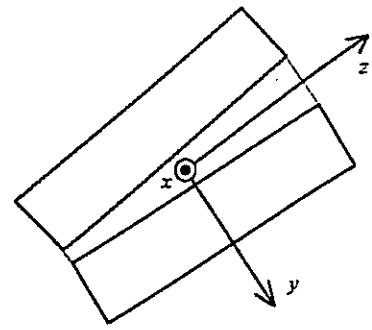

Fig. 2 Predominant elastic strain ε_y in a joint's cross section due to the relative rotation between the two beams

In the case of rectangular tapered section here considered, the factor of torsional rigidity can be expressed by the following equation:

$$I_{h,z}(x)=\beta\left(\frac{b}{a}\right)\left[\frac{a}{2}\left[\left(1\mp\frac{x}{c}\right)\right]\right]^3, \qquad \beta\left(\frac{b}{a}\rangle\rangle 1\right)=\frac{1}{3} \tag{3}$$

where a is the height of the beam's cross section (Figure 1).

The equations above indicate that the solution to the problem (eq. 1) is heavily dependent on the joint geometry (eq. 3).

At this point, the predominant stress field in the adhesive (equivalent to the applied torsional moment) can be determined by imposing rotational equilibrium of an infinitesimal beam element belonging to the bond area:

$$\sigma_y(x,z)=-\frac{z}{I_x^*}\frac{dM(x)}{dx}, \tag{4}$$

where z is the transverse coordinate (Figure 1). Equation (3) demonstrates that the increase in torsional moment occurring at the ends of the infinitesimal element considered is balanced by the normal stresses that the adhesive exerts on the beam.

The corresponding predominant stress field ε_y (Figure 2) can be written taking into account the constitutive law as:

$$\varepsilon_y(x,z)=\frac{\sigma_y(x,z)}{E_a^*}=-\frac{dM(x)}{dx}\frac{z}{E_a^*I_x^*}. \tag{5}$$

The compatibility equation permits to determine the joint's elastic strain, as relative rotation between the two beams' cross sections:

$$\Delta\theta(x)=\theta_2(x)-\theta_1(x)=-\frac{1}{K^*}\frac{dM(x)}{dx}. \tag{6}$$

Following a three dimensional finite element analysis, for non-tubular joints without tapered adherends, the mathematical approach was validated by comparing the predominant stress field given by equation (4) with that determined numerically, indicating an error on the stress peak of less than 4% [17].

The problem is bring back to determine section by section the torsional moment M solving the second order differential equation (1) in the case of tapered adherends (eq. (3)). The predominant stress and strain field in the adhesive and the joint's elastic strain can be obtained from equations (4), (5) and (6).

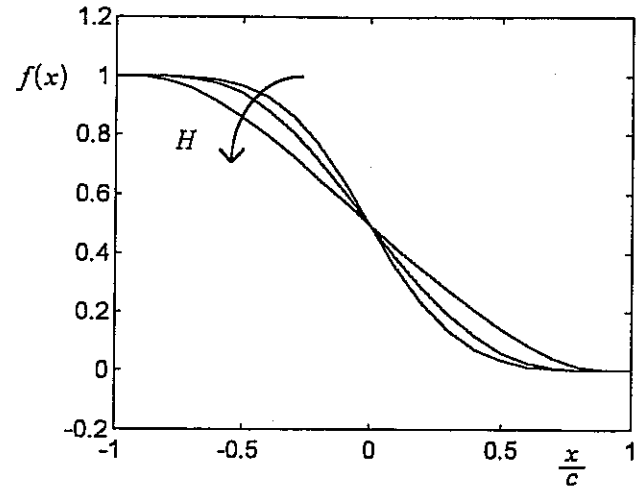

Fig. 3 Qualitative curve for $f(x)$ [torsional moments]

3 Closed form solution

The torsional moment $M(x)$ in the joint between the two beams can be written as:

$$\begin{aligned} M &= M_1(x) = M_t f(x) \\ M_2(x) &= M_t(1-f(x)) \end{aligned}, \qquad (7)$$

since the sum of the moments absorbed by the two elements must be equivalent to the applied torsional moment M_t for each section. Function $f(x)$ has as its domain the real range $[-c,+c]$ and, in order for the boundary conditions for the torsional moments (eq. (1)) to be satisfied, must be unity at the extreme left and zero at the extreme right of the domain. Moreover, as the two beams must be symmetrical as regards torsional moment and stress, the following conditions must also obtain:

$$\left.\frac{df(x)}{dx}\right|_x = \left.\frac{df(x)}{dx}\right|_{-x} \quad \forall x \qquad (8)$$
$$1-f(x)=f(-x)$$

Introducing the adimensional coordinate $x^* = x/c$, the equation (1) can be rewritten as:

$$\begin{aligned} &I_{t_1}(x^*)I_{t_2}(x^*)\frac{1}{c^2}\frac{d^2M(x^*)}{dx^{*2}}+ \\ &+\frac{K^*}{G}(I_{t_1}(x^*)+I_{t_2}(x^*))M(x^*)=. \\ &=-\frac{K^*M_t}{G}I_{t_1}(x^*) \end{aligned} \qquad (9)$$

Considering thin adherends ($b/a \gg 1$) and putting the equations (3) and (7) in (9), we obtain the following differential equation:

$$\begin{aligned} &H(1-3x^{*2}+3x^{*4}-x^{*6})\frac{d^2f(x^*)}{dx^{*2}}+ \\ &-(2+6x^{*2})f(x^*)=-(1-3x^*+3x^{*2}-x^{*3}). \\ &H=\frac{Ga^3b}{24K^*c^2} \end{aligned} \qquad (10)$$

The solution of this differential equation must satisfy (8) and can be found in power series form:

$$f(x^*)=\frac{1}{2}+\sum_{n=1}^{\infty}l_{2n-1}x^{*2n-1}. \qquad (11)$$

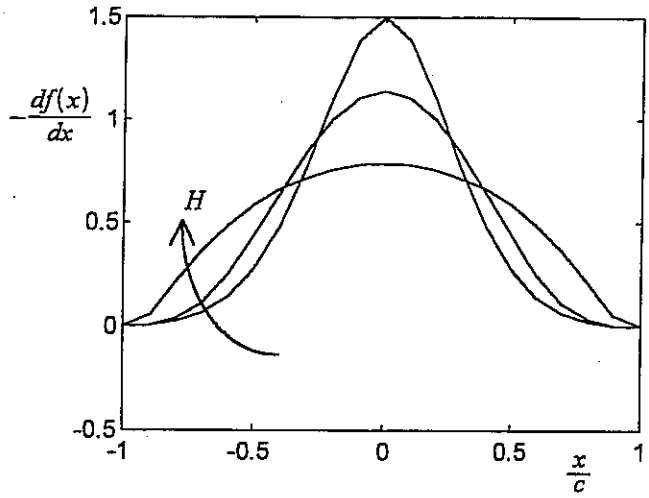

Fig. 4 Qualitative curve for $-\dfrac{df(x)}{dx}$ [relative rotations]

Through the principle of identity of polynomials, introducing this in (10) gives the following solution for the unknowns $l_{2n-1} = c_{2n-1} + d_{2n-1} l_1$, ($c_1$=0, d_1=1):

$$\begin{cases} c_3 = \dfrac{1}{2H} \\ d_3 = \dfrac{1}{3H} \end{cases}, \begin{cases} c_5 = \dfrac{10H+1}{20H^2} \\ d_5 = \dfrac{18H+1}{30H^2} \end{cases}$$

$$\begin{cases} c_7 = \dfrac{210H^2+70H+1}{420H^3} \\ d_7 = \dfrac{450H^2+78H+1}{630H^3} \end{cases} \quad (12)$$

$$(c,d)_{2n+1} = \frac{\frac{2}{H}+3(2n-1)(2n-2)}{(2n+1)(2n)}(c,d)_{2n-1} + \frac{\frac{6}{H}-3(2n-3)(2n-4)}{(2n+1)(2n)}(c,d)_{2n-3} + \frac{(2n-5)(2n-6)}{(2n+1)(2n)}(c,d)_{2n-5} \qquad \forall n \geq 4$$

Once the coefficients l_i are known as a function of l_1, the latter can be determined using the boundary condition $f(x^* = 1) = 0$:

$$l_1 = -\frac{\frac{1}{2}+\sum_{n=1}^{\infty} c_{2n-1}}{\sum_{n=1}^{\infty} d_{2n-1}}. \quad (13)$$

The stress σ_y acting on the first beam at the bonded surface is found from (4):

$$\sigma_y(x,z) = -\frac{M_t}{I_x^*}\frac{df(x)}{dx}z = -\frac{M_t}{I_x^* c}z\sum_{n=1}^{\infty}(2n-1)l_{2n-1}\left(\frac{x}{c}\right)^{2n-2}. \quad (14)$$

Focusing attention on the plane $z=b/2$ (critical), the mean and maximum value of the stresses are:

$$\sigma_{y_{med}} = \frac{1}{2c}\int_{-c}^{+c}\sigma_y(x)\,dx = \frac{M_t}{I_x^*}\frac{b}{4c} = \frac{3M_t}{b^2 c}, \qquad \sigma_{y_{max}} = \sigma_y(x=0) = -\frac{M_t}{I_x^*}\frac{b}{2c}l_1 \quad (15)$$

Consequently, a stress concentration factor can be defined as:

$$\lambda = \frac{\sigma_{y_{max}}}{\sigma_{y_{med}}} = -2l_1 = \frac{1+2\sum_{n=1}^{\infty} c_{2n-1}}{\sum_{n=1}^{\infty} d_{2n-1}}. \quad (16)$$

The rotation expressions of the two beams are reduced to integrals of known functions:

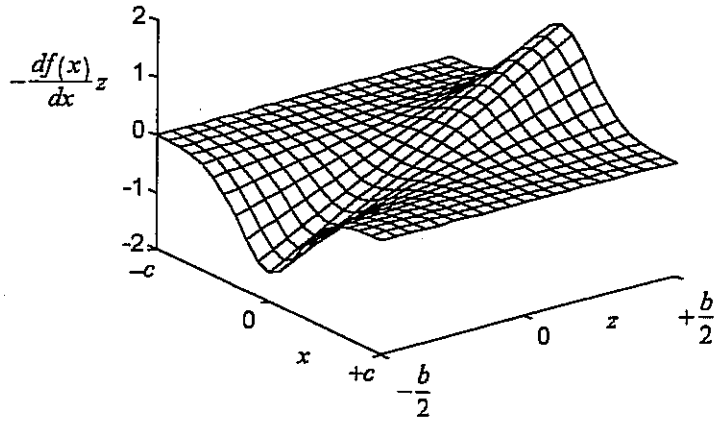

Fig. 5 Qualitative surface for $-\dfrac{df(x)}{dx}z$ [stresses and strains]

$$\theta_1(x)=\frac{M_t}{G}\int_{-c}^{x}\frac{f(t)}{I_{t_1}(t)}dt+\theta_1^0,$$
$$\theta_2(x)=\frac{M_t}{G}\int_{-c}^{x}\frac{(1-f(t))}{I_{t_2}(t)}dt+\theta_2^0 \qquad (17)$$

The compatibility equation (6) permits to determine the joint's elastic strain, as relative rotation. Fixing the reference system with $\theta_1^0=0$, the constant θ_2^0 is determined by inserting equations (17) into equation (6).
The results of the analysis described are presented in Figure 3,4,5. Figure 3 shows the curve for the function $f(x)$, governing torsional moment transmission in the joint, Figure 4 its derivative, which governs relative rotations and stresses in a constant z plane, and Figure 5 the stress field surface on the bond plane.
From equation (16) it is possible to obtain the value of the stress concentration factor. This value can be calculated in a approximated form as:

$$\lambda(H)\cong\frac{1+2\sum_{n=1}^{N}c_{2n-1}(H)}{\sum_{n=1}^{N}d_{2n-1}(H)}, \qquad (18)$$

where N is the finite number of terms considered. If N increases the series expansions converge and λ becomes N-independent.
Table 1 shows as λ (N-independent) changes with H.

4 Approximated solution

The H parameter for real joints assumes a value much smaller than 1 ($\approx 10^{-4}$). When H tends to zero the power series presented converges less rapidly and it is possible to have some numerical problems. In this case an approximated solution can be used. If we set H=0 in (10), the order of this differential equation becomes zero and the function $f(x)$ can be directly obtained. By applying equation (3), $f(x)$ assumes the physical meaning of relative torsional rigidity and coefficient of distribution:

$$f(x)=\frac{I_{t_1}(x)}{I_{t_1}(x)+I_{t_2}(x)}. \qquad (19)$$

As expected, this indicates that the stress field is extremely sensitive to external geometry. The simplified solution (19) satisfies the boundary conditions of the problem and the symmetrical conditions (8).
The correspondent torsional moment becomes:

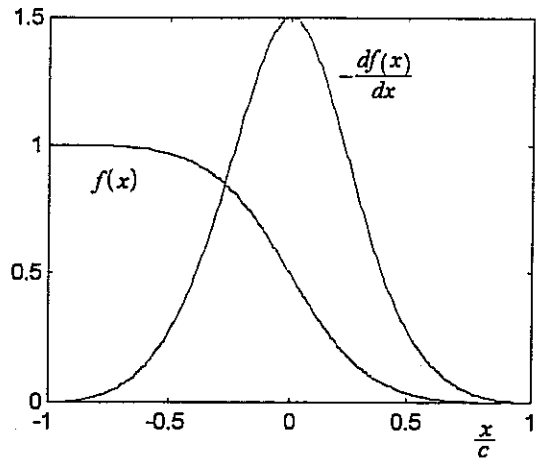

Fig. 6 Curves for $f(x)$ and its derivative ($H\to 0$)

$$M_1(x)=M_t-M_2(x)= \\ =\frac{1-3\left(\frac{x}{c}\right)+3\left(\frac{x}{c}\right)^2-\left(\frac{x}{c}\right)^3}{2+6\left(\frac{x}{c}\right)^2}\tilde{M}_t \quad (20)$$

The stress and strain predominant field become as follows:

$$\sigma_y(x,z)=\frac{(1-\nu_a)E_a}{(1+\nu_a)(1-2\nu_a)}\varepsilon_y(x,z)= \\ =\frac{18M_t}{cb^3}\left(\frac{\left(\frac{x}{c}\right)^2-1}{1+3\left(\frac{x}{c}\right)^2}\right)^2 z, \quad \lambda=\frac{\sigma_{y_{max}}}{\sigma_{y_{med}}}=3 \quad (21)$$

and the relative rotation between the two beams is:

$$\Delta\theta(x)=\theta_2(x)-\theta_1(x)= \\ =\frac{18(1+\nu_a)(1-2\nu_a)hM_t}{(1-\nu_a)E_ab^3}\left(\frac{\left(\frac{x}{c}\right)^2-1}{1+3\left(\frac{x}{c}\right)^2}\right)^2 \quad (22)$$

The torsional moment absorbed by the two beams (20) is proportional to the correspondent torsional stiffness (which in turn is proportional to the cube of the height). In any given section of the overlap, the top beam will thus tend to absorb almost all of the transmitted torsional moment, whose maximum variation will consequently take place at the centerline of the joint. The variation in torsional moment is balanced by the stresses (21) that the adhesive transmits to the beam. By contrast in the situation without tapered adherends joints (Pugno and Surace, 2000), these stresses will thus also be highest at the centerline. The stress concentration factor expressed is equal to 3. It is important to observe that it is an high upper bound for every practical value of H (Table 1). This stress concentration factor value is the same as we find in a linear elastic infinite plate with a circular hole subject to tension.

Figure 6 shows the function (19) and its derivative (torsional moment and stress/strain field in a z=const. plane or relative rotation). Clearly this approximated solution cannot be at the same time statically (equilibrium) and kinematically (compatibility) determinated: indeed the difference between equations (17), inserting eq. (19), is a constant value in disagreement with equation (22). In order to satisfy both equilibrium and compatibility equations the exact solution has to be considered.

$H=10^m$	m	-6	-5	-4	-3	-2	-1	0	1	2	3	4
	λ	3	2.999	2.997	2.977	2.824	2.279	1.567	1.163	1.033	1.004	1

Table 1 Stress concentration factor varying $H=10^m$ (real joint: m around -4)

5 Conclusions

A closed form solution for the problem of a non-tubular bonded joint with tapered adherends in torsion has been presented. The torsional moment transmission along the overlap, the predominant stress and strain fields in the adhesive and the joint's compliance have been obtained in series expansions. An approximated solution has been also presented.
It has been shown that, by contrast with the situation for conventional non-tubular joints, the stresses for tapered adherends will be surprisingly highest not at the ends of the adhesive layer but at its centerline. Finally, the correspondent stress concentration factor has been evaluated via the exact solution. It appears substantially constant and close to three.

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