Tubular bonded joint under torsion: theoretical analysis and optimization for uniform torsional strength

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Abstract: The paper analyses the problem of torsion in an adhesive bonded tubular joint. The constitutive, equilibrium and compatibility equations were used to obtain the stress field in the adhesive. The analysis confirms that the maximum stresses are attained at the ends of the adhesive and that the peak of maximum stress is reached at the end of the stiffer tube and does not tend to zero as the adhesive length approaches infinity.

A special type of tubular joint can be produced by modifying the joint profile, thus ensuring that the stress field in the adhesive is constant and thereby optimizing the tubular joint for uniform torsional strength. This result is of considerable practical utility and makes it possible to produce adhesive bonded joints which are both lighter and stronger under torsion. Finally, some suggestions for the joint design are presented.

Keywords: torsion, bonded joint, adhesive, tubular lap, optimization, uniform torsional strength

NOTATION

A, Z	auxiliary constants $= \sqrt{K^*(I_{p1} + I_{p2})/(GI_{p1}I_{p2})}$ and $I_{p1}/(I_{p1} + I_{p2})$ respectively
c	adhesive half-length
$\overline{c}$	x value of the point in the adhesive of the maximum shearing stress
C_1, C_2	integration constants $= e^{-Ac}/(e^{-2Ac} - e^{2Ac}) + Z(e^{Ac} - e^{-Ac})/(e^{-2Ac} - e^{2Ac})$ and $e^{Ac}/(e^{2Ac} - e^{-2Ac}) + Z(e^{-Ac} - e^{Ac})/(e^{2Ac} - e^{-2Ac})$ respectively
f	function of torsional moment transmission
G	shear modulus of the tube material
G_a	shear modulus of the adhesive material
h	adhesive thickness
I_p	polar moment of inertia of the two optimized tubes, outside the overlap zone
I_{pi}	polar moment of inertia of tube i
K^*	adhesive stiffness per unit length
M_f	applied torsional moment of failure
$M_{f,UTS}$	applied torsional moment of failure for the optimized joint
M_t	applied torsional moment
M_{tui}	torsional moment at a given cross-section through tube i
R	radius of the adhesive surface
R_1	external radius of the external tube
R_{1o}	external radius of the external optimized tube, outside the overlap zone
R_2	internal radius of the internal tube
R_{2o}	internal radius of the internal optimized tube, outside the overlap zone
x, y, z	axis coordinates
α, β	auxiliary constants $= R_{1o}/R$ and R_{2o}/R respectively
γ	shearing strain field in the adhesive
$\Delta\eta$	relative angular displacement through the adhesive thickness
θ_i	rotation at a given cross-section through tube i
θ_i^0	rotation at the initial cross-section of tube i
λ	stress concentration factor in the adhesive
λ^*	gain parameter for adhesive's stress levelling $= \tau_{min,max}/\tau_{max}$
τ	shearing stress field in the adhesive
τ_{adm}	admissible value of the shearing stress for the adhesive's material
$\tau_{adm\ tube}$	admissible value of the shearing stress for the tubes' material

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τ_f	failure value of the shearing stress for the adhesive's material
τ_{max}	maximum value of the shearing stress field in the adhesive
$\tau_{max\ tube}$	maximum value of the shearing stress field in the tubes
τ_{mean}	mean value of the shearing stress field in the adhesive
$\tau_{min,max}$	minimum value of the maximum values of the shearing stress field in the adhesive, varying c

1 INTRODUCTION

Bonded joints are very useful in structural elements. Indeed adhesives make it possible to join dissimilar and non-metallic components with savings also in terms of the weight of the resulting building structure. These advantages encourage the use of adhesive bonding for structures which until recently were joined using conventional techniques such as riveting, welding or threaded connections.

Considerable research in this area has been conducted regarding tubular structures [**1–25**], with the specific aspect of a non-tubular bonded joint under torsion having been analysed relatively recently [**21–32**]. Since the two pioneer papers [**1**, **2**], tubular joints subject to torsion have been studied from many different points of view. Theoretical approaches have been validated both experimentally [**3–6**] and numerically [**6–8**] and the importance of bonded joints in composite structures has been widely emphasized [**3**, **7**, **9**, **10**]. Fracture mechanics have been used to solve the problem of the joint's strength in the case of brittle collapse with static [**3**, **5**, **11**] or fatigue loading [**3**, **5**, **6**, **10**]. The non-linear and viscoelastic adhesive's behaviour has been considered in references [**6**] and [**11**] to [**15**] and recently also a dynamic analysis has been performed [**8**, **16**]. Specifically, in references [**7**] and [**17**], emphasis has been placed on the positive influence of tapering at the adherends' end on the stress peaks in the adhesive layer from numerical and theoretical points of view respectively; if the adherends are partially tapered, the stress peaks become lower.

The aim of this work is to determine theoretically a new kind of joint tapered profile, thus ensuring that the stress field is without stress peaks and perfectly constant, starting from the research documented in reference [**22**]. After a simple approach to determine the stress field in the adhesive, an *ad hoc* optimization for uniform torsional strength is presented. The same result, which would be of considerable practical utility since it could be used to produce joints which would be both lighter and stronger, has been developed recently also for non-tubular joints [**25**, **28**].

2 THEORETICAL ANALYSIS: ASSUMPTIONS AND VALIDITY

It is assumed that all three of the materials making up the joint (tubes and adhesive) are governed by an isotropic linear elastic law. While this is intuitive for tubes (which are typically metallic), this is not the case for the adhesive, which is more likely to exhibit typically non-linear behaviour. However, if the adhesive's film is considered to be under torsion and not subject to tension, according to the statistical theory of the rubber [**23**], its behaviour can be considered to be substantially linear elastic. The tubular bonded joint, consisting of two tubes perfectly circular and coaxial and the interposed adhesive's film (of very small thickness h), is considered to be subject to torque as shown in Fig. 1.

The tubes can be analysed using the 'technical theory of the beam' [**30**]. The results obtained for the Saint Venant solid on the basis of restrictive hypotheses, as regards both the geometry of the solid (rectilinear axis and constant cross-section) and the external loads (lateral surface not loaded and zero body forces), are usually extended in technical applications to cases in which these hypotheses are not satisfied. In any case it is required that the radius of curvature of the geometrical axis of the beam should be much greater than the characteristic dimensions of the cross-section, and that the cross-section should be only slightly variable.

It can also be assumed that only the shearing stress field $\tau_{r\theta}$ in the adhesive (considered constant over the film thickness) and $\tau_{x\theta}$ in the tubes are taken into account. The remaining components of the stress tensor in the adhesive layer and in the tubes are supposed to have negligible effects on joint deformations, if compared with $\tau_{r\theta}$ and $\tau_{x\theta}$ respectively, when a torque is applied to the tubes. The suitability of this assumption (i.e. the inaccuracy that it causes) is linked to the ratio of the overlap length to the tube thickness. It can be roughly evaluated by comparing the values of $\tau_{x\theta}$ in the tubes with the values of $\tau_{r\theta}$ in the adhesive [**14**].

The validity of the analysis proposed depends on correspondence with the assumptions above, which are relatively common in work of this kind. Under these conditions, it is possible to isolate an element of infinitesimal length d$x(-c \leqslant x \leqslant +c)$ belonging to the outer tube

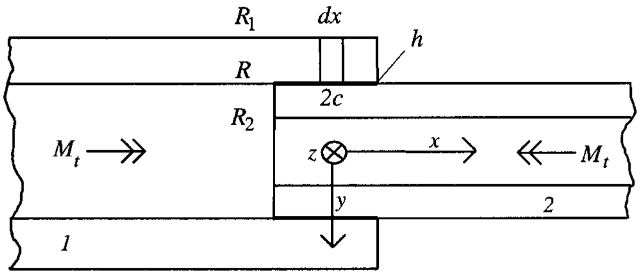

Fig. 1 Tubular joint under torsion

and impose rotational equilibrium around the axis of the centroid of the cross-sections parallel to the x axis (Fig. 2). The predominant stress field, equivalent to the applied torsional moment, in the adhesive can be obtained:

$$\tau(x) = -\frac{1}{2\pi R^2}\frac{\mathrm{d}M_{\mathrm{tu1}}(x)}{\mathrm{d}x} \tag{1}$$

where R is the adhesive surface's radius and $M_{\mathrm{tu1}}(x)$ the torsional moment in the outer tube in a generic x section. Equation (1) shows that the increase in torsional moment in the tube is balanced by the shearing stresses τ applied on the internal lateral surface of the external tube by the adhesive. The strain field γ in the adhesive can be obtained from the corresponding stress field [equation (1)]:

$$\gamma(x) = \frac{\tau(x)}{G_{\mathrm{a}}} = -\frac{1}{2\pi G_{\mathrm{a}} R^2}\frac{\mathrm{d}M_{\mathrm{tu1}}(x)}{\mathrm{d}x} \tag{2}$$

where G_{a} is the shear modulus of the adhesive.

The torsional moment $M_{\mathrm{tu}i}(x)$ in a generic section x of tube i can be written as

$$M_{\mathrm{tu1}}(x) = M_{\mathrm{t}} f(x) \tag{3a}$$

$$M_{\mathrm{tu2}}(x) = M_{\mathrm{t}}[1 - f(x)] \tag{3b}$$

as the sum of the moments absorbed by the two elements must be equivalent to the applied torsional moment M_{t} for every cross-section x. The function $f(x)$ has as its domain the real range $[-c, +c]$ and, in order for the boundary conditions for the torsional moments

$$M_{\mathrm{tu1}}(-c) = M_{\mathrm{t}}, \qquad M_{\mathrm{tu1}}(+c) = 0 \tag{4a}$$

$$M_{\mathrm{tu2}}(-c) = 0, \qquad M_{\mathrm{tu2}}(+c) = M_{\mathrm{t}} \tag{4b}$$

to be satisfied, must be unity at the extreme left and zero at the extreme right of the domain. The function $f(x)$, and thus the torsional moment absorbed by the two elements at the joint, can be found because of the compatibility established for the rotations of the two tube cross-sections. These rotations are expressed as follows:

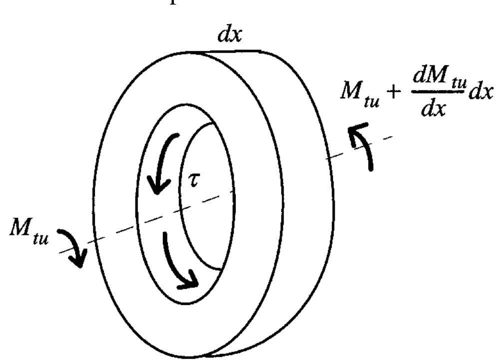

Fig. 2 Element of infinitesimal length

$$\theta_1(x) = \int_{-c}^{x} \frac{M_{\mathrm{tu1}}(t)}{GI_{\mathrm{p1}}}\,\mathrm{d}t + \theta_1^0 \tag{5a}$$

$$\theta_2(x) = \int_{-c}^{x} \frac{M_{\mathrm{tu2}}(t)}{GI_{\mathrm{p2}}}\,\mathrm{d}t + \theta_2^0 \tag{5b}$$

as G is the shear elastic modulus of the tubes, $I_{\mathrm{p}i}$ are their polar moments of inertia and θ_i^0 is the absolute rotation of the initial section ($x = -c$) of the tube i. Through an appropriate choice of reference system, $\theta_1^0 = 0$ always holds (rotations calculated starting from the strained configuration of the first tube's initial section).

The compatibility equation can be written noting as, after the joint's deformation, the relative angular displacement $\Delta\eta$ between two points of interfaces, internal tube–adhesive and adhesive–external tube, aligned on a same radial direction, must be the same if the tubes' relative rotation or the shearing adhesive's strain is considered (Fig. 3):

$$\Delta\eta = R[\theta_2(x) - \theta_1(x)] = R\,\Delta\theta(x) = h\gamma(x) \tag{6}$$

Substituting equation (2) into equation (6), the compatibility equation can be rewritten as

$$\frac{\mathrm{d}M_{\mathrm{tu1}}(x)}{\mathrm{d}x} = -K^*\Delta\theta(x), \qquad K^* = \frac{2\pi R^3 G_{\mathrm{a}}}{h} \tag{7}$$

where K^* is the adhesive's stiffness per unit length.

Inserting the rotation expressions (5) into the compatibility equation (7) gives the integro-differential relation

$$\frac{\mathrm{d}M_{\mathrm{tu1}}}{\mathrm{d}x}(x) = K^*\int_{-c}^{x}\left[\frac{M_{\mathrm{tu1}}(t)}{GI_{\mathrm{p1}}} - \frac{M_{\mathrm{tu2}}(t)}{GI_{\mathrm{p2}}}\right]\mathrm{d}t - K^*\theta_2^0 \tag{8}$$

This relation can be expressed by a single unknown $f(x)$; remembering equations (3), derivation gives the following second-order differential equation in $f(x)$:

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} - \frac{K^*(I_{\mathrm{p1}} + I_{\mathrm{p2}})}{GI_{\mathrm{p1}}I_{\mathrm{p2}}} f(x) = -\frac{K^*}{GI_{\mathrm{p2}}},$$

$$f(-c) = 1,\ f(c) = 0 \tag{9}$$

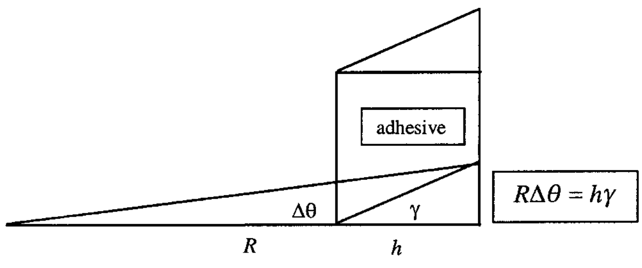

Fig. 3 Adhesive's cross-section; compatibility between adhesive and tubes

This differential equation, together with the boundary conditions shown below, make it possible to determine the torsional moment section by section at the overlap. The solution of equation (9) is

$$f(x) = C_1 e^{Ax} + C_2 e^{-Ax} + Z \tag{10}$$

with

$$A = \sqrt{\frac{K^*(I_{p1} + I_{p2})}{GI_{p1}I_{p2}}}, \qquad Z = \frac{I_{p1}}{I_{p1} + I_{p2}} \tag{11}$$

The constants C_1 and C_2 can be obtained from the boundary conditions as

$$C_1 = \frac{e^{-Ac}}{e^{-2Ac} - e^{2Ac}} + Z\frac{e^{Ac} - e^{-Ac}}{e^{-2Ac} - e^{2Ac}} \tag{12a}$$

$$C_2 = \frac{e^{Ac}}{e^{2Ac} - e^{-2Ac}} + Z\frac{e^{-Ac} - e^{Ac}}{e^{2Ac} - e^{-2Ac}} \tag{12b}$$

The function (10) governs the torsional moments [equation (3)], its derivative

$$\frac{df(x)}{dx} = A(C_1 e^{Ax} - C_2\, e^{-Ax}) \tag{13}$$

the stress [equation (1)] and strain [equation (2)] field in the adhesive and its integral

$$\int_{-c}^{x} f(t)\,dt = Z(x+c) + \frac{1}{A} \times (C_1 e^{Ax} - C_2\, e^{-Ax} - C_1 e^{-Ac} + C_2\, e^{Ac}) \tag{14}$$

the absolute rotations of the two tubes [equation (5)]. The joint elastic strain as relative rotations can be obtained directly from the compatibility equation (7). Comparing the differences between equations (5b) and (5a) with the same obtained by equation (7) makes it possible to determine constant θ_2^0, once the reference system has been established with $\theta_1^0 = 0$:

$$\theta_2^0 = \frac{M_t A}{K^*}(C_2 e^{Ac} - C_1 e^{-Ac}) \tag{15}$$

Figure 4 shows the curves for the function $f(x)$ governing torsional moment transmission in the joint, while Fig. 5 illustrates its derivative, which governs relative rotations or stresses and strains varying the two constants represented by equation (11). The maximum stresses are reached at the ends of the adhesive and the higher stress peak appears at the end of the stiffer tube. When the stiffnesses of the two tubes are equal ($Z = \frac{1}{2}$), the stress field becomes lower and symmetric.

3 JOINT DESIGN: CONSTANT-HEIGHT PROFILE

Considering equation (1) it is possible to define a stress concentration factor which indicates the extent to which maximum stress departs from the mean. The higher stress peak appears at the end of the stiffer tube:

$$\tau_{max} = \tau(x = \overline{c}) = \frac{M_t A}{2\pi R^2}(-C_1 e^{A\overline{c}} + C_2\, e^{-A\overline{c}}),$$

$$\overline{c} = \begin{cases} -c, & 0 < Z < \frac{1}{2} \\ c, & \frac{1}{2} \leq Z < 1 \end{cases} \tag{16}$$

The mean value of the stress field is

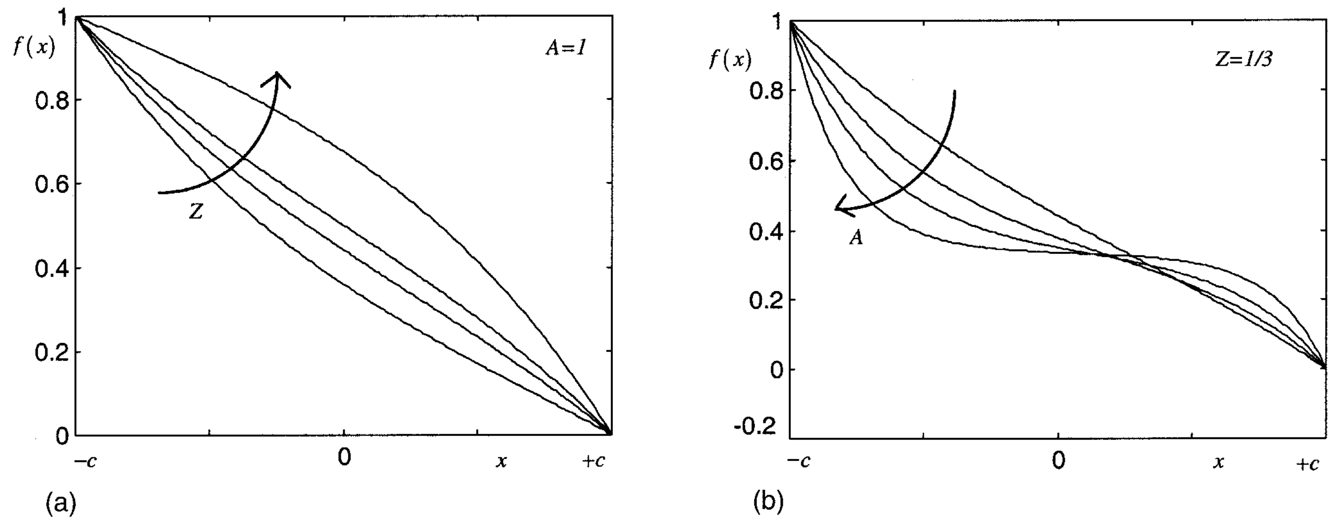

Fig. 4 Qualitative curves for $f(x)$ (a) by varying Z with $A =$ constant and (b) by varying A with $Z =$ constant

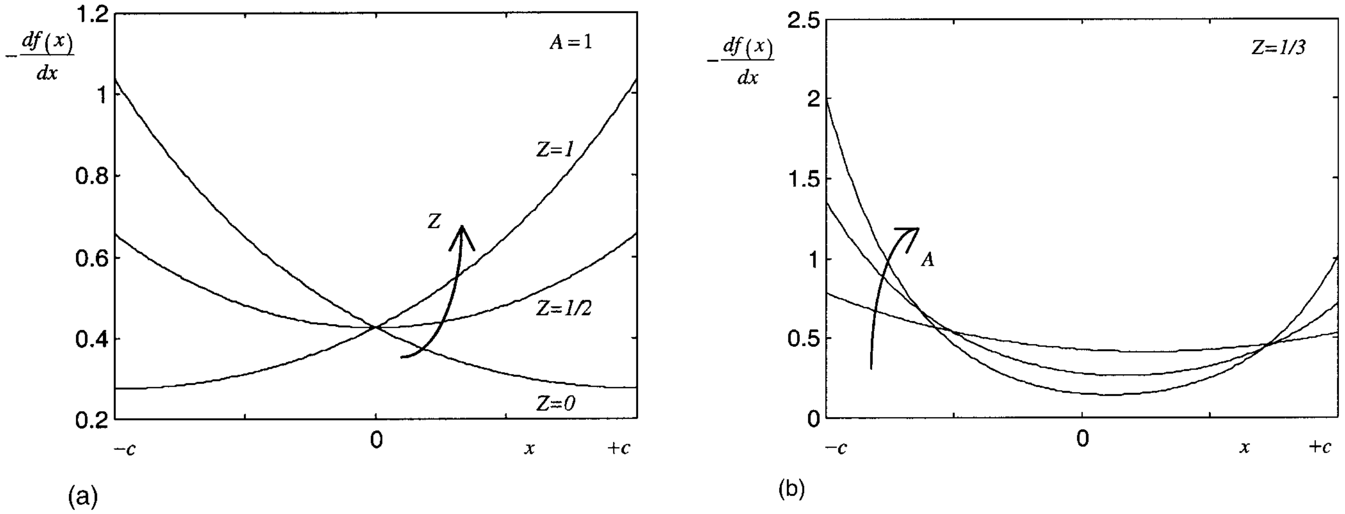

Fig. 5 Qualitative curves for $-\,\mathrm{d}f(x)/\mathrm{d}x$ (a) by varying Z with A = constant and (b) by varying A with Z = constant

$$\tau_{\text{mean}} = \frac{1}{2c}\int_{-c}^{+c} \tau(x)\,\mathrm{d}x = \frac{M_\mathrm{t}}{4\pi R^2 c} \tag{17}$$

Consequently, the stress concentration factor is given by

$$\lambda = \frac{\tau_{\max}}{\tau_{\text{mean}}} = 2Ac(-C_1 \mathrm{e}^{A\bar{c}} + C_2\,\mathrm{e}^{-A\bar{c}}) \tag{18}$$

Of importance is the gain parameter λ^*, i.e. the index of the gain in maximum stress levelling, which can be obtained by increasing the bond length. In this context, it should be noted that, as the bond length tends to infinity, the maximum stress tends asymptotically to a minimum non-zero value:

$$\tau_{\min,\max} = \lim_{c\to\infty} \tau_{\max} = \frac{M_\mathrm{t} ZA}{2\pi R^2} \tag{19}$$

The gain parameter can thus be defined as

$$\lambda^*(Ac) = \frac{\tau_{\min,\max}}{\tau_{\max}} = \frac{Z}{(-C_1 \mathrm{e}^{A\bar{c}} + C_2\,\mathrm{e}^{-A\bar{c}})} \tag{20}$$

and must be as close to unity as is compatible with the need for a compact joint. Under this assumption the stress concentration factor, prudently overestimated, is detailed as follows:

$$\lambda \approx 2ZAc \qquad \text{for } \lambda^* \approx 1 \tag{21}$$

Figure 6 shows that the gain parameter λ^* presents little variation after a certain value of the non-dimensional parameter Ac (≈ 3); consequently, further increases in bond length are pointless for the torsional strength. Furthermore Z must be equal to $\frac{1}{2}$ (same polar moment of inertia for the two tubes) to have a symmetric stress field. Under these assumptions the stress concentration factor appears to be close to 3, an often-used value in elastic problems.

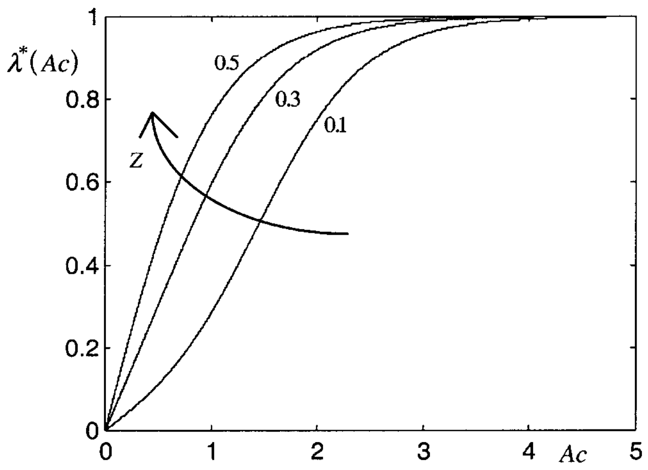

Fig. 6 Gain parameter $\lambda^*(Ac)$

4 OPTIMIZATION FOR THE UNIFORM TORSIONAL STRENGTH (UTS)

In order to obtain a unit value for the stress concentration factor given by equation (18) it is possible to modify the joint profile. This is achieved by chamfering the edges, which are in any case not involved in an eventual stress flow induced by a tensile load. The procedure used is the reverse of that employed for a joint of known geometry: rather than starting from the geometry in order to determine the stress field, the procedure starts with the stress field and determines the geometry capable of ensuring it.

In order to make the stress field constant, it must be independent of the x coordinate. In other words, as shown by equation (1), the torsional moment must be linear along the joint x axis:

$$f(x) = \frac{1}{2} - \frac{x}{2c} \tag{22}$$

Inserting equation (22) in equation (9) yields the following relation, which defines the geometry of a uniform torsional strength (UTS) adhesive bonded joint:

$$\frac{I_{p2}(x)}{I_{p1}(x)} = \frac{c+x}{c-x} \tag{23}$$

Substituting into equation (23) the expressions for the polar moment of inertia of circular tubes gives

$$\frac{R^4 - R_2^4(x)}{R_1^4(x) - R^4} = \frac{c+x}{c-x} \tag{24}$$

From equation (24), it can be seen that the height of the terminal portion of the tubes must be zero; in fact,

$$R_1(x=c) = R, \qquad R_2(x=-c) = R \tag{25}$$

In order to identify families of optimized bonded joint profiles, it is necessary to introduce a function of some kind, $R_1(x)$ for example, which respects the previous condition [equation (25)]. Then, through equation (24), the function $R_2(x)$ (which must be real in all its domain) whereby the joint has uniform torsional strength can be determined.

Although the number of possible shapes which satisfy the relations indicated above is infinite, the following additional condition must be considered in order to obtain the solution entailing tubes with a symmetric polar moment of inertia section by section:

$$I_{p1}(x) = I_{p2}(-x) \tag{26}$$

which permits there to be identical polar moments of inertia, I_p, for the two tubes out of the bonded area. Equations (23) and (26) are satisfied by the following expressions for the profiles of the two tubes which define the geometry of the joint optimized for UTS:

$$R_1(x) = \sqrt[4]{R^4 + \frac{c-x}{2c}(R_{1o}^4 - R^4)} = \sqrt[4]{R^4 + \frac{1-x/c}{\pi} I_p} \tag{27a}$$

$$R_2(x) = \sqrt[4]{R^4 - \frac{c+x}{2c}(R^4 - R_{2o}^4)} = \sqrt[4]{R^4 - \frac{1+x/c}{\pi} I_p} \tag{27b}$$

where

$$R_{1o} = R_1(x=-c), \qquad R_{2o} = R_2(x=c) \tag{28}$$

and

$$R^4 - R_{2o}^4 = R_{1o}^4 - R^4 = \frac{2I_p}{\pi},$$
$$0 \leqslant R_{2o} < R \Rightarrow R < R_{1o} \leqslant \sqrt[4]{2}R \tag{29}$$

Equation (29) shows also the limits for the two radii R_{io}.

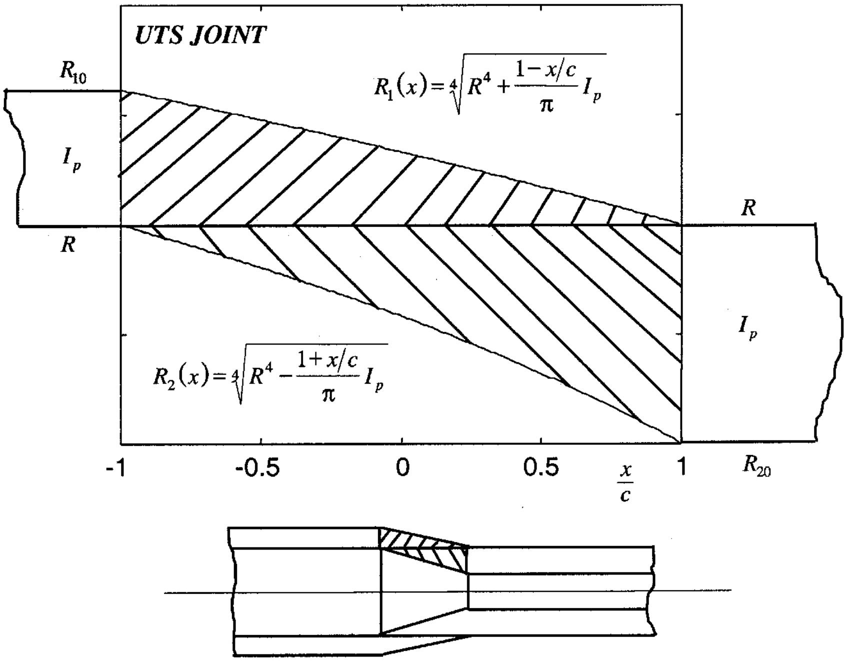

Fig. 7 UTS joint's profile

The profile thus obtained [equation (27)] is shown in Fig. 7.

The gain of a UTS joint compared with the conventional type (constant-height profile) can be defined as the ratio of the maximum torsional moments that can be borne by the UTS and conventional joints, once a certain collapse phenomenon has been assumed. If it is supposed that joint collapse takes place when the maximum shearing stress is equal to an ultimate value τ_f, characteristic of the adhesive, the following relations show how the UTS joint's gain over the conventional type coincides with its stress peak parameter [equation (13)]:

$$M_t = M_f \qquad \text{when } \tau_{max} = \tau_f, \quad \frac{M_{f,UTS}}{M_f} \equiv \lambda \tag{30}$$

The ratio of the maximum torsional moment that can be transmitted by the UTS joint to that transmitted by the joint with constant height coincides exactly with its ratio of the maximum value to the mean value in the adhesive's stress field.

5 JOINT DESIGN: UTS PROFILE

The maximum shearing stress due to a torsional moment in a section of the tube is obtained at the external radius of the external tube, out of the bonded area. This value must be lower than a characteristic admissible value for the tube's material:

$$\tau_{max,\, tube} = \frac{2}{\pi} \frac{M_t}{R_{1o}^4 - R^4} R_{1o} \leqslant \tau_{adm,\, tube} \tag{31}$$

Referring to equation (29) it is found that

$$R_{1o} = \alpha R, \qquad R_{2o} = \beta R, \qquad 0 \leqslant \beta < 1, \; \alpha = \sqrt[4]{2 - \beta^4} \tag{32}$$

and inequality (31) becomes

$$R \geqslant \sqrt[3]{\frac{2(2-\beta^4)^{1/4}}{\pi(1-\beta^4)} \frac{M_t}{\tau_{adm,\, tube}}} \tag{33}$$

Inequalities (32) and (33) are the sufficient and necessary conditions for existence of the UTS joint. If they are satisfied, the corresponding UTS joint exists and, if they are not satisfied, the UTS joint does not exist.

Fixing the torsional moment and the shearing admissible stress for the tube material, the joint's radial dimension R_{1o} can be obtained from equations (33) and (32):

$$R_{1o} \geqslant \sqrt[3]{\frac{2(2-\beta^4)}{\pi(1-\beta^4)} \frac{M_t}{\tau_{adm,\, tube}}} \tag{34}$$

If $\beta = 0$, the joint's radial dimension R_{1o} is a minimum but the corresponding joint weight (proportional to $\alpha^2 - \beta^2 = \sqrt{2-\beta^4} - \beta^2$) is a maximum; the opposite occurs for $\beta = 1$.

The adhesive length can be obtained as a function of the torsional moment assuming a characteristic value for the admissible shearing stress of the adhesive [equation (17)]:

$$c \geqslant \frac{M_t}{4\pi R^2 \tau_{adm}} \tag{35}$$

For the joint design, β must satisfy inequality (32), R inequality (33), α, R_{1o} and R_{2o} equalities (32) and c inequality (35). The joint profile can be obtained from equations (27) or approximately (but the safety coefficients increase) with a linear tapering of the adherends.

If inequalities (33) and (35) becomes equalities in the tube and in the adhesive, the shearing stresses equal their admissible values. In this context, it should be noted that, as the bond length tends to infinity, the maximum stress [equal to the mean value expressed by equation (17)] tends asymptotically to a minimum zero value. This is a very important behaviour of the UTS joint because theoretically, differently from a non-tapered joint, the adhesive can withstand every torsional moment, simply modifying its length surface. This upper bound of torque, increasing the adhesive length, for non-tapered adherends and supposing that $I_{p1} = I_{p2} = I_p$, can be obtained from equation (19):

$$M_f(c \to \infty) = \sqrt{\frac{4\pi R h I_p G}{G_a \tau_f}} \tag{36}$$

and is infinity for the optimized joint.

However, it is important to note that adhesive bonded joints could be susceptible to brittle collapse. In order to take advantage of the UTS joint geometry it is essential that appropriate technological measures be introduced to ensure that joint collapse cannot involve mechanical fracture phenomena.

6 CONCLUSIONS

The theory developed in this article indicates that the reported optimal profile for uniform strength, even if purely theoretical, could give useful guidelines to designers of tubular bonded joints under torsion. The hypotheses are substantially of isotropic linear elastic law for tubes, with only slightly variable cross-sections, and adhesive, with small thicknesses. This optimal shape would permit both reduced weight and increased strength (excluding mechanical fracture phenomena). The constant shearing stress field in the bond would enable the adhesive to withstand every torsional moment by simply modifying its length surface.

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REFERENCES

1 **Goland, M.** and **Reissner, E.** The stresses in cemented joints. *J. Appl. Mechanics,* 1944, **11**, 17–27.
2 **Lubkin, J. L.** and **Reissner, E.** Stress distribution and design data for adhesive lap joints between circular tubes. *Trans. ASME*, 1956, **78**, 1213–1221.
3 **Reedy, E. D.** and **Guess, T. R.** Composite-to-metal tubular lap joints strength and fatigue resistance. *Int. J. Fracture*, 1993, **63**, 351–367.
4 **Kim, K. S., Kim, W. T., Lee, D. G.** and **Jun, E. J.** Optimal tubular adhesive-bonded lap joint of the carbon-fiber epoxy composite shaft. *Composite Structs*, 1992, **21**, 163–176.
5 **Gent, A. N.** and **Yeoh, O. H.** Failure loads for model adhesive joints subjected to tension, compression or torsion. *J. Mater. Sci.*, 1982, **17**, 1713–1722.
6 **Choi, J. H.** and **Lee, D. G.** The torque transmission capabilities of the adhesively-bonded tubular single lap joint and the double lap joint. *J. Adhesion,* 1994, **44**, 197–212.
7 **Hipol, P. J.** Analysis and optimization of a tubular lap joint subjected to torsion. *J. Composite Mater.*, 1984, **18**, 298–311.
8 **Rao, M. D.** and **Zhou, H.** Vibration and damping of a bonded tubular lap joint. *J. Sound Vibr.*, 1994, **178**, 577–590.
9 **Graves, S. R.** and **Adams, D. F.** Analysis of a bonded joint in a composite tube subjected to torsion. *J. Composite Mater.*, 1981, **15**, 211–224.
10 **Nayebhashemi, H., Rossettos, J. N.** and **Melo, A. P.** Multiaxial fatigue life evaluation of tubular adhesively bonded joints. *Int. J. Adhesion Adhes.*, 1997, **17**, 55–63.
11 **Lee, S. J.** and **Lee, D. G.** Development of a failure model for the adhesively bonded tubular single lap joint. *J. Adhesion*, 1992, **40**, 1–14.
12 **Lee, S. J.** and **Lee, D. G.** An iterative solution for the torque transmission capability of adhesively-bonded tubular single lap joints with nonlinear shear properties. *J. Adhesion*, 1995, **53**, 217–227.
13 **Alwar, R. S.** and **Nagaraja, Y. R.** Viscoelastic analysis of an adhesive tubular joint. *J. Adhesion*, 1976, **8**, 79–92.
14 **Medri, G.** Viscoelastic analysis of adhesive bonded lap joints between tubes under torsion. *Trans. ASME, J. Vibr., Acoust., Stress Reliability Des.,* 1988, **110**, 384–388.
15 **Zhou, H. M.** and **Rao, M. D.** Viscoelastic analysis of bonded tubular joints under torsion. *Int. J. Solids Structs*, 1993, **30**, 2199–2211.
16 **Ko, T. C., Lin, C. C.** and **Chu, R. C.** Vibration of bonded laminated lap-joint plates using adhesive interface elements. *J. Sound Vibr.*, 1995, **4**, 567–583.
17 **Chen, D.** and **Cheng, S.** Torsional stresses in tubular lap joints with tapered adherends. *J. Engng Mechanics Div., Am. Soc. Civ. Engrs*, 1992, **118**, 1962–1973.
18 **Chen, D.** and **Cheng, S.** Torsional stress in tubular lap joints. *Int. J. Solids Structs*, 1992, **29**, 845–853.
19 **Adams, R. D.** and **Peppiatt, N. A.** Stress analysis of adhesive bonded tubular lap joints. *J. Adhesion*, 1977, **9**, 1–18.
20 **Chon, C. T.** Analysis of tubular lap joint in torsion. *J. Composite Mater.*, 1982, **16**, 268–284.
21 **Medri, G.** Il calcolo delle tensioni nell'adesivo in giunti tra tubi sollecitati da momento torcente. *Ingegn. Mecc.*, 1977, **7–8**, 247–251.
22 **Pugno, N.** and **Surace, G.** Ottimizzazione di un giunto incollato tubular-lap progettato ad uniforme resistenza a torsione. In Proceedings of the 27th AIAS, Perugia, Italy, 1998, pp. 1043–1052.
23 **Treloar, L. R. G.** *The Physics of Rubber Elasticity,* 3rd edition, 1975 (Clarendon, Oxford).
24 **Pugno, N.** Non-tubular bonded joint under torsion. PhD thesis, Department of Structural Engineering, Politecnico di Torino, Torino, Italy, 1998.
25 **Pugno, N.** Optimizing a non-tubular adhesive bonded joint for uniform torsional strength. *Int. J. Mater. Product Technol.*, 1999, **14**, 476–487.
26 **Pugno, N.** and **Surace, G.** Theoretical tensional analysis of a single lap bonded joint subjected to torsion. In Proceedings of the 7th NMCM, Hihg Tatras, Slovak Republic, 1998, pp. 189–195.
27 **Pugno, N.** and **Surace, G.** Analisi teorica tensionale di un giunto incollato double-lap soggetto a torsione. In Proceedings of the 27th AIAS, Perugia, Italy, 1998, pp. 1053–1062.
28 **Pugno, N.** and **Surace, G.** Non tubular bonded joint under torsion: theory and numerical validation. *Structural Engng Mechanics,* 2000, **10**, 125–138.
29 **Pugno, N.** Closed form solution for a non tubular bonded joint with tapered adherends under torsion. *Computer Structs,* 1999 (submitted).
30 **Carpinteri, A.** *Structural Mechanics—A Unified Approach*, 1997 (E. and F. N. Spon, London).