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The centenary of Griffith's theory

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The year 2021 marks the 100th anniversary of Griffith's Fracture Theory. Many scholars regard Griffith as the father of the linear elastic fracture mechanics. Here, Nicola Pugno—who received the Griffith Medal and Prize in 2017—reflects on some Griffith's based approaches and quantized-related extensions of the original theory, as he has orally presented this year during his Plenary Opening Lecture at the quadrennial 25th International Congress of Theoretical and Applied Mechanics.

In 1921, Sir Alan Arnold Griffith, an English mechanical engineer, published his seminal paper describing the theory of Linear Elastic Fracture Mechanics (LEFM)[1] in the Philosophical Transactions of the Royal Society (Figure 1). Today, 100 years later, this elegant theory shows new implications even in the standardization of the mechanical property measurements and comparisons.

The ultimate failure of materials and structures limits our current technologies and lives, and frequently, this failure is rooted in a phenomenon known as fracture—the sudden and abrupt cracking/breaking of a hard object or material such as metals and glass, but also bones and rock. Understanding fracture mechanics in several disciplines, from nano-engineering to earthquake engineering including medicine (e.g., bone fracture), is thus vital. Predicting the maximum loads a material could take while avoiding fracture was a critical mechanics challenge.

Prior to Griffith, the initiation of fracture, however, was not easy to predict. Consider a simple linear elastic sheet. The nominal stress in a sheet can be calculated as the applied force divided by its cross-sectional area. Undergraduate engineering students learn that stress is simply load over an area. The tensile strength of the sheet, however, cannot be simply predicted assuming failure when the stress reaches the critical strength value, as originally thought. This is due to the fundamental role played by defects—always present in materials and structures and thus governing their behaviors—in analogy to what happens in human beings.

Let us assume the presence of a tiny crack in the sheet. According to LEFM, failure is governed not only by the stress, σ, but a combination between the stress and the crack length a, formulated as a geometry- and load-dependent stress intensity factor, K (i.e., for the classical Griffith case of linear elastic infinite plate with a far field tensile stress σ and a perpendicular crack of length $2a$, then $K = \sigma\sqrt{\pi a}$). Failure via fracture can then be reliably predicted when K reaches a critical value, the so-called fracture toughness K_C of the material, which dictates the onset of crack propagation. Thus, for the Griffith case, the fracture strength (stress) is predicted as $\sigma_C = K_C/\sqrt{\pi a}$ (the reader could verify this dependence with a sheet of paper inserting cuts with different lengths).

This result naturally emerges from the Griffith's idea of an energy balance during crack growth, involving elastic energy, external work, and dissipated energy spent in the crack extension: the so-called energy release rate, the opposite of the variation with respect to the crack surface area of the total potential energy (elastic energy minus external work), has to reach a critical value for crack propagation, i.e., the fracture energy (per unit area) of the material. The energy release rate is proportional to the square of the stress intensity factor divided by the Young's modulus (for a given fracture mode)[2] and, similarly, their critical values (fracture energy and toughness) so the two criteria are equivalent. This theory has thus revealed the mystery and dramatic role of defects in the strength of materials.

In order to solve some remaining limitations of LEFM, including a paradox of *infinite strength for vanishing crack length*, we extended LEFM by introducing quantized fracture mechanics (QFM[3]), substituting differentials with finite differences in Griffith's energy balance. This corresponds to considering the onset of crack propagation not when the energy release rate reaches a critical value, but when its mean value along a fracture quantum of crack surface area ΔA—the previously introduced finite difference—reaches the material fracture energy (simply put, crack growth is not continuous, e.g. at smaller and smaller scales, but reaches characteristic discrete steps of finite length, or fracture quantum, during propagation). Equivalently, it is not the stress intensity factor $K \equiv K_{LEFM}$ that must reach K_C for crack propagation, but instead the square root of the mean value of the square

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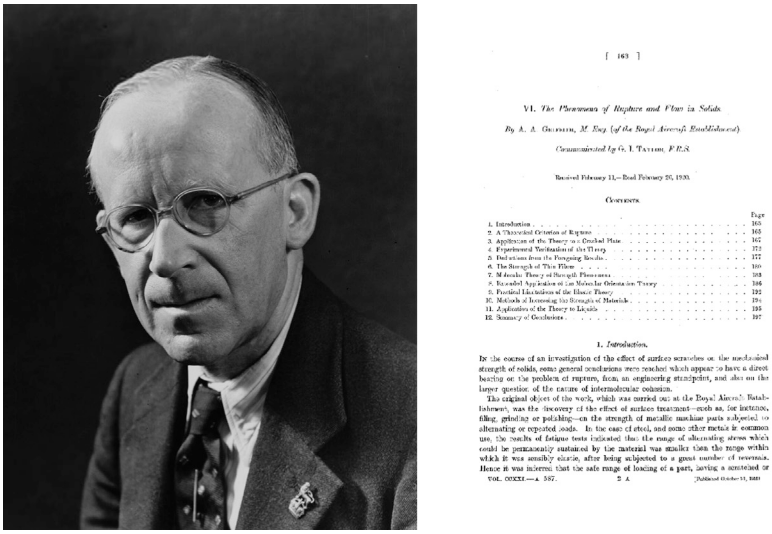
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VI. *The Phenomena of Rupture and Flow in Solids.*

By A. A. Griffith, *M. Eng.* (*of the Royal Aircraft Establishment*).

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1. *Introduction.*

In the course of an investigation of the effect of surface scratches on the mechanical strength of solids, some general conclusions were reached which appear to have a direct bearing on the problem of rupture, from an engineering standpoint, and also on the larger question of the nature of intermolecular cohesion.

The original object of the work, which was carried out at the Royal Aircraft Establishment, was the discovery of the effect of surface treatment—such as, for instance, filing, grinding or polishing—on the strength of metallic machine parts subjected to alternating or repeated loads. In the case of steel, and some other metals in common use, the results of fatigue tests indicated that the range of alternating stress which could be permanently sustained by the material was smaller than the range within which it was sensibly elastic, after being subjected to a great number of reversals. Hence it was inferred that the safe range of loading of a part, having a scratched or

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Figure 1. Griffith and his seminal paper

Left: Sir A.A. Griffith. Right: The first page of his pioneering paper published 100 years ago: The phenomena of rupture and flow in solids. Philosophical Transactions of the Royal Society of London. 221, 163–198, 1921.

of the stress intensity factor along a fracture quantum, which is by definition the quantized stress intensity factor K_{QFM} (for example, for the Griffith's plate the QFM strength prediction –from $K_{QFM}=K_C$– is $\sigma_C=\frac{K_C}{\sqrt{\pi(a+q)}}$, $q=\frac{\Delta A}{2t}$ and t is plate thickness). This theory is thus based on the removal of the hypothesis of continuous crack growth in the Griffith's energy balance, i.e., on the existence of a quantum of energy dissipation/crack advancement. In dynamic fracture, and considering the existence of a quantum of action, the mean value must be considered also along a quantum of time Δt, thus defining the dynamic quantized stress intensity factor K_{DQFM}[4] in dynamic quantized fracture mechanics (DQFM).

This exponent of 2 of the stress intensity factor appearing in D/QFM can in principle be generalized to a positive real number α, a unification originally proposed in reference 5,[5] thus proposing a Griffith/generalized dynamic quantized fracture mechanics (GQFM), predicting crack propagation according to:

$$K_{GQFM}^{(\alpha,\Delta A,\Delta t)} \equiv \sqrt[\alpha]{\frac{1}{\Delta A\Delta t}\int_{t-\Delta t}^{t}\int_{A}^{A+\Delta A} K(A,t)^{\alpha}\,dA\,dt} = K_c$$

(Equation 1)

Note that $K_{GQFM}^{(\alpha,\Delta A=0,\Delta t=0)} = K_{LEFM}$, $K_{GQFM}^{(\alpha=2,\Delta A,\Delta t=0)} = K_{QFM}$, $K_{GQFM}^{(\alpha=2,\Delta A,\Delta t)} = K_{DQFM}$ and $K_{GQFM}^{(\alpha=1,\Delta A=0,\Delta t)} = K_{ITFM}$, where K_{ITFM} is the equivalent stress intensity factor according to the incubation time based fracture mechanics approach.[6] Accordingly, all these limiting-case theories, which have been proved to capture experimental observations that LEFM cannot describe, are recovered and thus the correspondence principle is satisfied, as expected in the limit of vanishing fracture energy and action quanta. Equation 1 represents (by definition) an extension of all the mentioned theories of fracture.

The stress intensity factor (Equation 1) could also extend stress intensity factor based laws such as those of fatigue, as proved for $\alpha=2$ to capture both long and short crack behaviors even in fatigue (thus from the Wöhler to the Paris regimes).[7] With the criterion (Equation 1), so-called R-curve behaviors and strain-rate effects emerge naturally, rendering respectively fracture toughness no longer/less crack-length[3] and strain-rate[4] dependent, and thus a more realistic material property (e.g., using $K_{LEFM}=K_C'$ in order to obtain the same result of the more realistic prediction $K_{QFM}=K_C$ one must assume $K_C'=\frac{K_C}{\sqrt{1+q/a}}$ thus a crack-length dependent fracture toughness tending to a constant value only for long crack length; this is usually observed in experiments). Accordingly, this could also help in the standardization of mechanical property measurements and comparisons, for example using the classical Ashby's plots.

Indeed Asbhy's plots, plots where materials are compared in terms of 2 mechanical properties of interest, e.g., fracture toughness and strength, are fundamental for material selection. As is well known, simply assuming in the Griffith's theory a crack length $a \propto l^{\beta}$, where l is the structural size, would result in a scaling law for the strength of the power-law type $\sigma_C \propto l^{-\beta/2}$, as similarly predicted by Weibull's statistics or by other interpretations, e.g., fractal geometry.[8] We do not wish to comment here about the details of size-effect laws, but rather to emphasize that with the advent of nanotechnology, it is not infrequent to see papers reporting the fabrication of a new material using an Ashby's plot to prove its superior characteristics, even if tested at a smaller size-scale. The need to adopt different Ashby's plots at the micro- and nano-scales has already been highlighted.[9] Recently, we have proposed[5] to use 3D Ashby's plots for a multiscale, thus complete, representation.

As a simple example, we treat the case reported in Figure 2, where nanotube bundles are considered according to Koziol et al.[10] The Asbhy's plot presented by the authors was inevitably

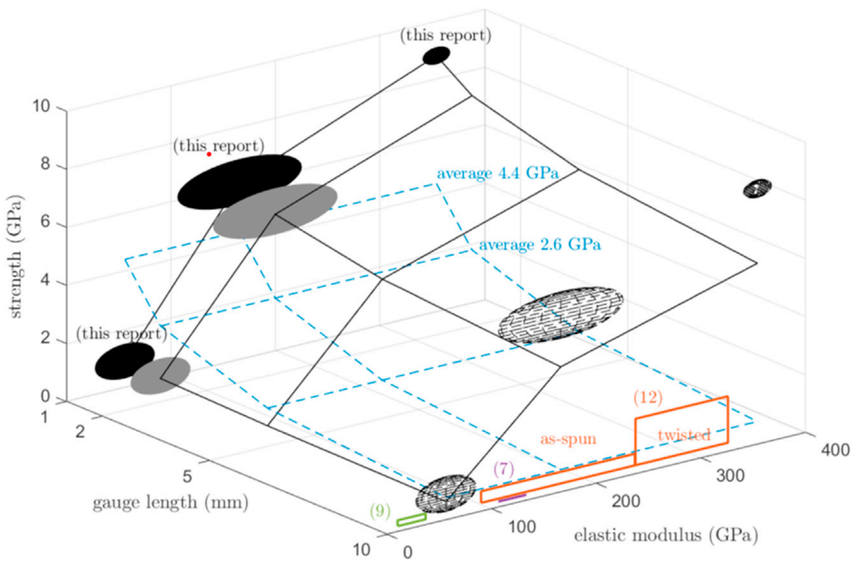

Figure 2. A multiscale 3D Asbhy's plot

Originally proposed in ref.[5], the modified Ashby's plot depicts strength versus elastic modulus versus gauge length. The black ellipses represent the strength versus elastic modulus data collected by the authors[10] corresponding to samples with 1-mm gauge length (their Figure 4B). The overall average of this data is 4.4 GPa, while we have computed a total average of 2.6 GPa for the data at 2-mm gauge length (their Figure 3A). The corresponding scaling law, considering in a first approximation the independence of the elastic modulus, is of the type $\sigma_C \propto l^{-\beta/2}$ with $\beta/2 \cong 0.75$ (blue dashed lines). This scaling law predicts strength values comparable to those taken from the literature (green, purple or red areas, 10-mm gauge length). When considering the low- and high-strength regions (peaks in their Figure 3A), the average for the ellipse centered at a strength of 6.2 GPa for 1-mm gauge length falls to 5.6 GPa for a 2-mm gauge length, thus giving a scaling law with $\beta/2 \cong 0.15$ (black solid lines). The dashed ellipses are the projections at 10-mm gauge length of those at 1-mm gauge length, as reported in their reference[10], whereas our mean or maximal predictions at 10-mm are those reported with the blue dashed or back continuous lines, respectively, as previously described.

classical, i.e., 2D, thus without considering the role of the bundle/gauge length. A 3D Asbhy's plot, with a new axis corresponding to the size-scale, here gauge-length, could thus be a better choice for comparison with other experiments, as described in Figure 2. The majority of the experiments reported for comparison in Koziol et al.[10] where performed at a gauge length of 1 cm, which we have considered in the 3D plot. We have also reported their experiments,[10] not in the same plane but in their correct positions (1-mm or 2-mm gauge length planes). According to the same authors' experiments at different gauge lengths and assuming the simple mentioned scaling law of $\sigma_C \propto l^{-\beta/2}$, we could predict mean or maximal strength values expected at larger gauge-lengths, including 10-mm, as reported respectively with the blue dashed or black solid lines. Thus, the other references used for comparison were nearly equally exceptional in terms of attained strength values.

The need for generalized Griffith's theories, scaling laws and multiscale 3D Asbhy's plots is thus evident if we wish to move toward a standardization in material property measurements and comparisons.

Concluding—100 years after its development—the celebrated Griffith's theory has still something to teach us.

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