

# The commemoration of Leonardo da Vinci

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To commemorate the 500th anniversary of the death of Leonardo da Vinci occurring in 2019, I propose the following manuscript, in the form of a virtual interview to prof. Giuseppe Maria Pugno (GMP), who commemorated the 500th anniversary of his birth in 1952 [1] (Fig. 1). The interview, although of a virtual nature, due to the two generations that separate the interviewer (the undersigned, NMP) and the interviewee, is nevertheless based on real answers given by GMP in his writings on Leonardo and the mechanics of solids and structures [2, 3]. The interview is thus confined to this discipline without wishing to touch upon Leonardo's contributions in other disciplines, or in mechanics itself, e.g. in fluid mechanics or applied mechanics, let alone in the field of art, painting and sculpture. Other interesting ideas can be found in

GMP's writings on Leonardo and machines [4], on Leonardo and hydraulics [5] and on the development of scientific thought in the 200 years from Dante Alighieri to Leonardo [6], as well as in essays by other authors on Leonardo, e.g. see [7–16] and references therein. In the last question and answer, interviewer and interviewee will exchange roles to take a look at Leonardo's legacy for a twenty-first century researcher, with only a few examples [17–28].

NMP: Prof. Giuseppe Maria Pugno, we know that the Atlantic Code (“maggior codice Vinciano”) risked being lost in numerous occasions, and that it was the dispersed and then found again material that gave rise to the Minor Codes. Among these, which ones are the most interesting for Mechanics and in particular for Structural Mechanics?

GMP: It is the “Codice Arundelliano”, the “Codicetto” on bird flight, the so-called A and B manuscripts and, in some lesser respects, the “Codice Trivulziano”.

NMP: Is it fair to say that Leonardo had a clear idea of Statics? For example, commenting the case of a load suspended on two wires (see the original drawing in Man. A sheet 47 verso), he correctly points out that the tensions in the wires only depend on their inclination. Or that he had also grasped the theorems at the base of the solution of hyperstatic structures, which the aforementioned example could become if a third thread were added in the same plane, or the

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virtual works principle (as a definition of internal work) and that of minimum potential energy?

GMP: Both, though not stated, were certainly intuitively understood by Leonardo. For example, Leonardo analyzes the case of two pulleys, as I have schematized in Fig. 2 (original drawing in Cod. Atl. 104 v.b.). Here we find, albeit in a particular case (of real forces and displacements), the virtual works theorem. Leonardo himself discusses applications like a sort of pincer that I named “Leonardo’s nutcracker”, schematically shown in Fig. 3 (original drawing in Code Atl. sheet 153 recto a), and comments on it in various ways (for example “E quella cosa che manco resiste, da minor potenza fia remossa, a più a lungo moto fia continuata”, i.e. “And that thing that resists less, is moved by a smaller force but for a longer displacement”, Cod. Atl. sheet 104 verso b). About 100 years later Stevin stated Leonardo’s proposition in the following form: “Ut spatium agentis ad spatium patientis, sic potentia patientis, ad potentiam agentis”. However, we must go as far as Bernoulli (1717), Fourier and Lagrange (1800), to see Leonardo’s proposition expressed in its most general form and demonstrated in the most complete way.

NMP: And coming back to the energy minimum?

GMP: Leonardo clearly felt and wrote that a body subjected to forces is lead to its natural state (congruent as well as balanced), obeying a precise condition of minimum: (“Ogni azione naturale—he writes—è generata dalla natura nel più briève modo che trovar si possa”: “Every natural action is generated by nature in the most brief way that we may find”, Cod. Atl. sheet 112 verso a). However, some 350 years were still to pass before the statement was made in the form of the Menabrea theorem.

NMP: To connect the two worlds of equilibrium and congruence and thus solve the hyperstatic problem, however, one needs constitutive laws. Had Leonardo guessed Hooke’s law, first stated by the latter in 1678 with the latin phrase “Ut tensio sic vis”?

GMP: Leonardo clearly also understood this law stating clearly the proportionality between the forces and deformations produced by them (he writes: “Questa molla che ha la forza per 200, sta ferma con peso di 200; ma se tu leverai un minimo di peso, essa

molla si drizzerà tanto che si pareggerà con la loro resistenza. E tal proporzione avrà tal moto con la lunghezza della molla quale ha il peso che si tolse col suo rimanente”, i.e. “This spring that has the strength of 200, remains undeformed with a weight of 200; but should you take away a minimum of weight, it will spring up so far as to be balanced by their resistance. And this proportion will have such motion with the length of the spring that has the weight that was taken away with its remnant”, Cod. Atl. sheet 110 verso b).

NMP: The forces applied externally on solids are transferred internally along lines called “isostatic”. Had Leonardo conjectured their existence?

GMP: Certainly, as is evident when Leonardo suggests “a way to constrain beams so they do not bend”, as I have schematized in Fig. 4 (original drawing in Cod. Atl. sheet 9 recto b), a connection that forces the beams to work cooperatively after one is lowered.

NMP: Leonardo seems therefore to have understood all the laws of hyperstatic structures. Had he also somehow surmised the de Saint–Venant hypothesis?

GMP: Without a doubt. In the study of normal stresses, Leonardo immediately perceives that the contact system does not have any influence on the shape of the stress diagram at a certain distance from the extremes. Not only this, but also that the diagram has a constant magnitude, as shown in Fig. 5. Considering a rope stretched by a centre load, that is to say along the geometric axis of the rope, he writes: “Ogni parte della grossezza d’essa corda sarà da esso peso ugualmente carica”, i.e. “Every part of the thickness of the rope will be of equally loaded” (Cod. Atl. sheet 153 recto a). About 300 years later, Adhémar-Jean-Claude Barré de Saint–Venant stated his principle.

NMP: Leonardo was therefore ready to design truss structures.

GMP: And this he did. As an example, consider his “movable bridge”, intended for rapid assembly and disassembly, which is schematized in Fig. 6 (see original drawing in Cod. Atl. sheet 312 recto a). It is essentially the “Fink” truss, from the name of the American civil engineer who realized it, about three and a half centuries later.

## La commemorazione di Leonardo da Vinci ad iniziativa del Comune di Torino e della Società degli Ingegneri e degli Architetti

Il 13 maggio 1952, per iniziativa del Comune e della Società degli Ingegneri e degli Architetti in Torino è stato commemorato il quinto centenario della nascita di Leonardo da Vinci.

Nel Salone del Senato in Palazzo Madama erano presenti S. Em. Cardinale Arcivescovo Maurilio Fossati, il Prefetto, il Primo Presidente della Corte d'Appello, S. E. il vescovo di Sebaste ausiliare di S. Em., e le più alte Autorità Civili e Militari del Piemonte.

Sul palco prendevano posto assieme all'oratore ufficiale, il Sindaco di Torino avv. Peyron, i due Vice Presidenti della Società prof. Dalla Verde ed arch. Grassi, rappresentanti gli enti promotori della celebrazione.

Preso la parola, il Sindaco ha espresso l'omaggio della Città alla figura di Leonardo « che sempre più giganteggia nella storia della Patria ».

Oratore ufficiale era il Presidente della Società degli Ingegneri e degli Architetti di Torino e Preside della Facoltà d'architettura del Politecnico, prof. ing. Giuseppe Maria Pugno.

Il conferenziere ha iniziato il suo dire con un rapido sguardo allo stato delle conoscenze scientifiche ai tempi di Dante mostrando come molte notizie faticosamente conquistate specialmente dai Greci ed Alessandrini fossero andate miseramente disperse. Ha poi fatto notare come da Dante incominci e con Leonardo possa riguardarsi affermato stabilmente il processo evolutivo che trasformò il pensiero scientifico dalla impostazione quasi rivelata e tradizionalista, alla spregiudicatezza di adattarsi alle manifestazioni naturali purchè rigorosamente controllate, accennando che con Leonardo appunto prende forma la nuova metodologia cui si legherà più tardi il nome del Galilei. Per sottolineare una tale trasformazione nel pensiero scientifico, vengono presi in considerazione soltanto pochi ma importanti aspetti della storia del progresso nei due secoli dal '300 al '500: esperienza, ordine naturale, scienze matematiche, moto, volo, luce. Specialmente nel trattare della luce, Leonardo si rivela nella pienezza del multiforme ingegno e pare che davanti a lui, quasi mosso alla conquista del mondo, cadano e si sfascino i baluardi tradizionalmente innalzantisi tra materia ed energia, tra materia e spirito. Ma dove il raffronto tra Dante e Leonardo più luminosamente si manifesta è nella loro concezione di Dio. Sembra che Dante già pos-

segga Dio, mentre Leonardo porgendo l'occhio, l'orecchio, la mente, il cuore a tutto ciò che sta attorno a lui, se lo conquista. Ambedue questi sommi intelletti finiscono per giungere allo stesso punto e perfino alla stessa definizione della Divinità: l'Amore, che per Dante è la forza immensa che muove il sole e l'altre stelle e per Leonardo « vincit omnia », e ad esso tutto deve cedere.

Applausi protrattisi a lungo e vive congratulazioni hanno salutato la conclusione della conferenza, la quale, trattando del mondo filosofico, letterario e scientifico leonardiano, ha messo in luce l'animo del grande Vinciano sotto un aspetto del tutto originale e forse mai presentato nelle numerose commemorazioni centenarie, tenute in quest'anno, di Leonardo.

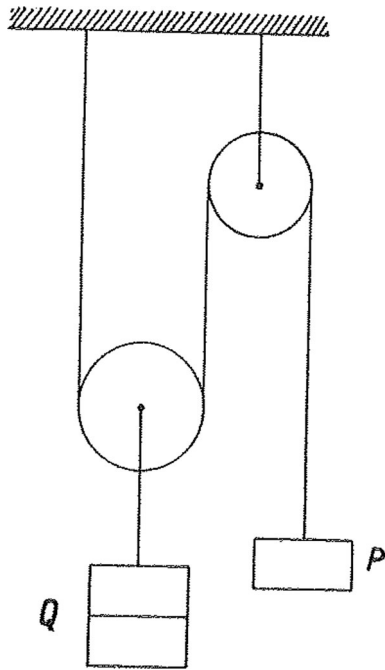
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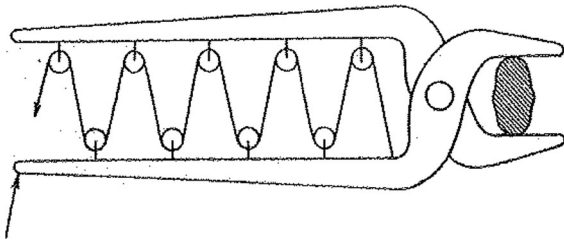
ATTI E RASSEGNA TECNICA DELLA SOCIETÀ DEGLI INGEGNERI E DEGLI ARCHITETTI IN TORINO - NUOVA SERIE - ANNO 6 - N. 5 - MAGGIO 1952 **139**

**Fig. 1** The page of the “Atti e rassegna tecnica della società degli ingegneri e degli architetti in Torino”, from May 1952, which reported the news of the Commemoration of Leonardo for

the 500 years from his birth by Prof. Giuseppe Maria Pugno [1] and introduced by the mayor of Turin, Amedeo Peyron



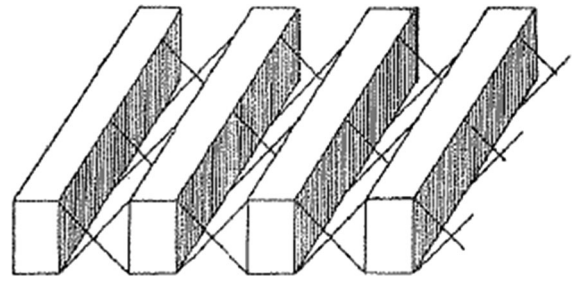
**Fig. 2** Towards the virtual works theorem



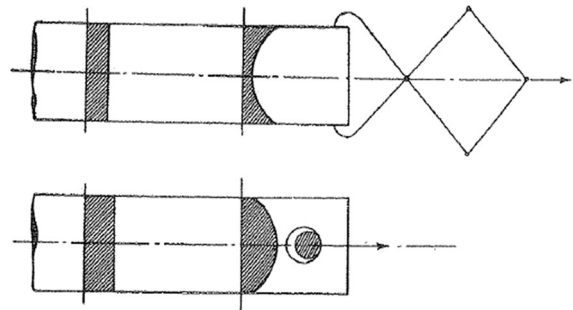
**Fig. 3** Leonardo's nutcracker

NMP: The comprehension of normal compressive stresses and of the related Euler load, and in particular of how this is inversely proportional to the square of the length of the prismatic solid with constant section loaded with an axial compressive tip force, must have been an insurmountable problem at the time of Leonardo. Am I right?

GMP: Leonardo undoubtedly understood the importance of the length of a solid subjected to an axial compressive tip load and examined two solids of equal section, one ten times longer than the other, as I have schematically indicated in Fig. 7 (original drawing in Cod. Atl. sheet 152 recto b). Here, unfortunately, Leonardo indicates the critical loads as 100 and 1000, and not the correct value of 10,000 for the latter.



**Fig. 4** “A way to constrain beams so they do not bend”



**Fig. 5** Inference of the De Saint–Venant principle

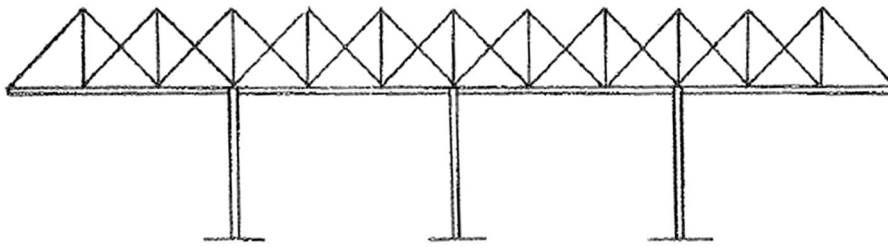
Later, however, he not only demonstrates that he understood the importance of the length of the solid, but also of the area of its cross section. With a complicated analysis, he describes what is indicated in simplified form in Fig. 8, deriving how one single beam supports a critical load of  $P$ , four distinct ones a load  $4P$ , but if joined together they support a load  $16P$ , here in perfect agreement with Euler's formula.

Leonardo often returns to these matters in Man. A sheet 3 verso 45 verso 46 recto, and it is interesting to observe the torment of his manuscript corrections on this subject: he used to mark or cross out in two different ways the propositions he had recognized as incorrect and those on which he still had doubts and on which he wished to return.

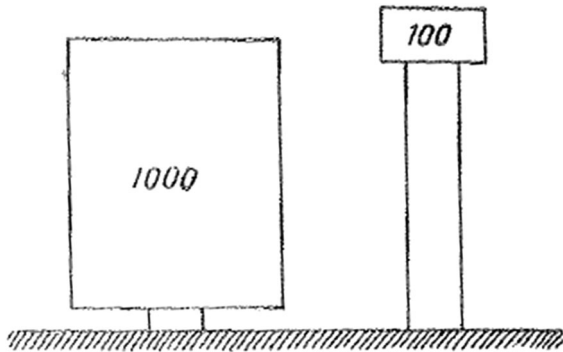
NMP: Leonardo also studied bending both in terms of tension and deformation.

GMP: In bending Leonardo mistakenly indicates a direct proportionality between the maximum bending moment that can be supported by a beam with a rectangular section and each of the dimensions of the section. Although he incorrectly formulates the law, he nevertheless intends and applies it correctly, recognizing the convenience in firmly wedging

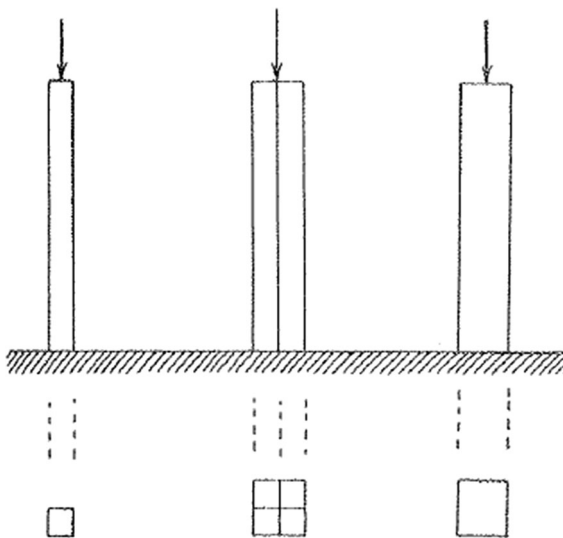




**Fig. 6** Leonardo's, or Fink's truss

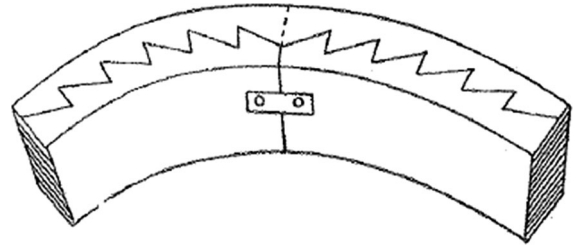


**Fig. 7** Leonardo's work on the influence of the length of a solid on its critical load when loaded with an axial compressive tip load



**Fig. 8** Leonardo's understanding of the influence of cross section of a solid subjected to an axial compressive tip load

together two beams to increase their overall height and strength, for example by adopting the arrangement schematically shown in Fig. 9 (see original drawing in Cod. Atl. sheet 344 verso a).



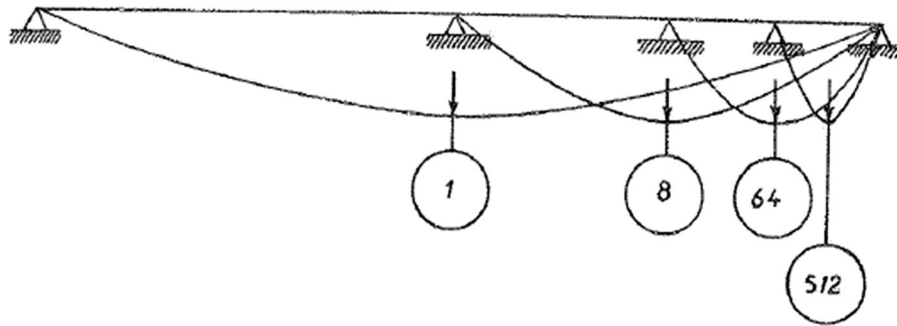
**Fig. 9** Beams wedged together to increase the overall bending strength

Concerning the deflection of the bent beams, Leonardo understands how this scales with the cube of the length of the beam. He considers the case of centrally loaded beams and with recursively halved lengths and searches for the values of the applied forces to generate the same deflection, as I show in Fig. 10 and as also Leonardo clearly indicates he understands (with the drawing in Cod. Atl. sheet 211 recto b).

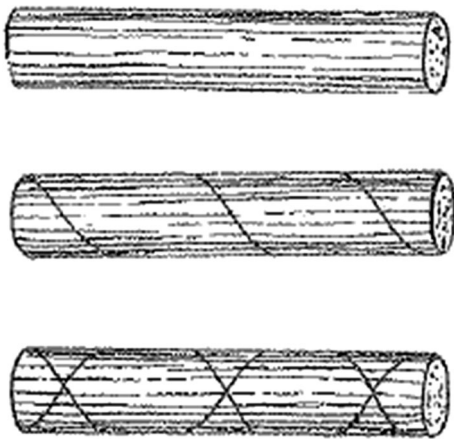
Moreover, he declares himself ready to say which weight  $P'$  it is necessary to apply at a given point on the beam to subject it to the same deflection produced by a known weight applied in the centre, demonstrating a greater understanding of Betti's theorem than Maxwell and Rayleigh themselves.

NMP: Leonardo also demonstrates to have understood torsion.

GMP: He deals with it by treating the case of a bundle of wickers in simple contact with each other and calls it “fasciculo disarmato”: “unreinforced bundle”. He proposes a reinforcement to withstand torsion in a single direction (“fasciculo armato sol per un verso”: “reinforced bundle only in one direction”) or both (“fasciculo ad un moto per due versi”), as shown in Fig. 11 (see original drawing in Cod. Atl. sheet 139 recto c). He therefore clearly shows that he has understood how isostatic lines in torsion are



**Fig. 10** Understanding beam deflection



**Fig. 11** Understanding torsional reinforcement

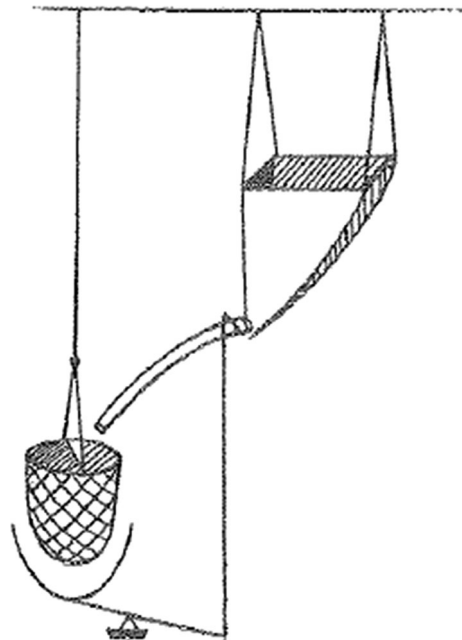
arranged in a spiral at 45 degrees with respect to the axis of the solid.

NMP: Leonardo also deals with compound stresses, in terms of normal stress and bending, and also with solids with uniform resistance to normal stress or bending, proving he understood these phenomena. What about strength of materials?

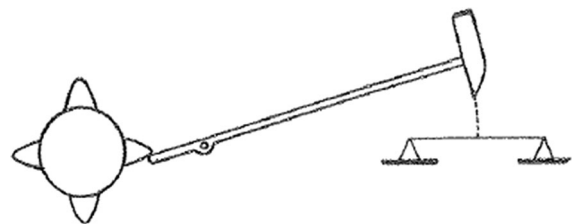
GMP: Leonardo, recognizing the importance of tensile tests for the characterization of the strength of materials, proposes a machine for their realization, as shown in Fig. 12 (original drawing in Cod. Atl. sheet 82 recto b) and also for repeated impact tests, as shown in Fig. 13 (original drawing in Cod. Atl. sheet 21 recto a).

NMP: Among Leonardo's impressive and well-known machines, we find a certain number for military applications, but we must remember that Leonardo was not a man of war but of peace.

GMP: Leonardo considered war as madness, and wrote: "E tu uomo, che consideri in questa mia fatica



**Fig. 12** Machine for traction tests



**Fig. 13** Machine for repeated impacts

l'opere mirabili della natura, se giudicherai essere cosa nefanda il distruggerla, or pensa essere una nefandissima cosa il tôrre la vita all'uomo; se questa sua composizione ti pare di meraviglioso artificio, pensa questa essere nulla rispetto all'anima che in tal

architettura abita”, i.e.: “And you, man, who considers in this work of mine the admirable works of nature, if you judge it to be vile to destroy it, now think it the vilest thing to take away life from man; if this creation seems to you a wonderful artifice, think it as being nothing compared to the soul that lives in such architecture”. Like Dante, Leonardo also comes to the definition of God as Love, which for the first is “Love that moves the sun and other stars” (Divine Comedy, Par. XXXIII, 145) and for the latter is “Amor qui omnia vincit, et nos cedamus amori”, i.e. “Love that conquers all, to which we must yield” (Cod. Atl. sheet 273 recto a).

NMP: Today’s researchers, politicians and humanity in general must also grasp this conquest of Leonardo in depth. One last question...

GMP: Now it is I who would like to ask you a question, about today’s researchers. What is Leonardo’s legacy for a 21st century researcher?

NMP: Leonardo has also paved the way for studies inspired by Nature. His flying machines are the most obvious example. Today we have technology that he did not have access to, which allows us to observe natural materials and fabricate bio-inspired ones with nanoscopic resolution and precision. Just as he could be fascinated by a limpet, today we can extract a microscopic sample from one of its teeth and discover the strongest material in Nature [17]. Inspired by spider web junctions, we can produce the toughest fibres in the world, exploiting slip knots that dissipate energy by friction [18]. Observing spiders, we have discovered the mechanisms with which it could lift much greater weights than itself (e.g. ideally in a couple of months even a 80 kg man) [19]. Leonardo had already guessed the laws that govern friction, but he certainly could not investigate how it can vary as a result of surface micro-patterning, which we can now design through numerical simulations and realize using laser texturing [20]. We can design composites exploiting single atomic sheets such as graphene [21], inspired by nacre [22] and replicating the hierarchical organization of natural materials that we can currently analyse in detail using advanced microscopy tools. With so-called metamaterials, today we can control the propagation of elastic waves, with applications from seismic shields to energy harvesting [23]. The examples could continue, considering the most classic lotus effect

for self-cleaning or gecko-effect for smart adhesion. The first is related to the modification of the wettability of a surface due to its roughness, possibly hierarchical, which increases its intrinsic wettability characteristics, thus making surfaces from chemically hydrophobic to super-hydrophobic (and often self-cleaning) such as the lotus (or vice versa from hydrophilic to super-hydrophilic) [24]. The second is related to the increase in van der Waals [25] adhesion resulting from the miniaturization of the contacts, the adhesive resistance being proportional to their total perimeter and not to their contact area [26]. Looking at nature at the microscopic level, mechanical elements have been discovered that man was thought to have invented, like the gear wheels themselves [27]. Finally, by feeding them nanomaterials such as nanotubes and graphene, we verified that spiders can produce a superior-strength silk compared to the already strong and very tough natural counterpart [28], with a concept of “bionicomposite” that surpasses the very inspiration from Nature.

*Mechanics—writes Leonardo—is the paradise of mathematical sciences, because that is where it bears fruit.*

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