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Time-scale effects during damage evolution: A fractal approach based on acoustic emission

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Abstract. In this paper we propose a fractal theory for the prediction of the *time-effects* on the damage evolution in cracking solids. By means of the Acoustic Emission (AE) technique, we have analysed the evolution of damage in several structures by an extensive experimental analysis in time. This technique permits to estimate the amount of energy released during fracture propagation and to obtain information on the criticality of the ongoing process. Theory and experiments agree closely. Moreover, based on fractal concepts, we have formulated a multiscale criterion to predict the damage progress in concrete structural elements. According to this method, the damage level of a structure can be estimated from AE data of a reference specimen extracted from the structure and tested up to failure. By monitoring the fracture propagation by AE, it is therefore possible to evaluate the damage level of a structure as well as the time corresponding to final collapse. Consequently, the life-time predictions of monitored solid structures can be estimated: as an example we have analysed the evolution of damage in two piers supporting a viaduct along an Italian highway built in the 1950s.

Keywords: Acoustic emission monitoring, structural assessment, damage mechanics, crack growth, life prediction

1. Introduction

The evaluation of safety and reliability for reinforced concrete structures, like bridges and viaducts, represents a complex task at the cutting edge of scientific research. Due to these reasons, the diagnosis and monitoring techniques are assuming an increasing importance in the evaluation of structural conditions and reliability. Among these methods, the nondestructive methodology based on Acoustic Emission (AE) proves to be very effective [1–3].

Some applications of AE technique to construction monitoring are described by Carpinteri and Lacidogna [4,5]. In addition, strong *space-effects* are clearly observed on energy density dissipated during fragmentation. Recently, a multiscale energy dissipation process has been shown to take place in fragmentation, from a theoretical and fractal viewpoint as proposed by Carpinteri and Pugno [6,7]. This fractal theory takes into account the multiscale character of energy dissipation and its strong space effects. Such a approach for the space-scaling of the energy density has been experimentally verified by AE technique [8,9]. Here we focus the attention on the complementary effects, related to the *time-effects*. The understanding of the *space–time-effects* make it possible to introduce a useful energetic damage parameter for structural assessment based on a correlation between AE activity in a structure

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and the corresponding activity recorded on a small specimen extracted from the structure and tested to failure. Moreover, by our findings on space–time-effects, the safety of structures undergoing damage and degradation processes can be efficiently evaluated in exercise conditions and *in situ*.

2. The acoustic emission technique

Monitoring a structure by means of the AE technique, it proves possible to detect the occurrence and evolution of stress-induced cracks. Cracking, in fact, is accompanied by the emission of elastic waves which propagate within the bulk of the material. These waves can be received and recorded by transducers applied to the surface of the structural elements. This technique, originally used to detect cracks and plastic deformations in metals, has been extended to studies and research in the field of rocks and can be used for diagnosing structural damage phenomena [2]. In AE monitoring, piezoelectric (PZT) sensors are generally used, thereby exploiting the capabilities of certain crystals to produce electric signals every time they are subjected to a mechanical stress. As a rule, the amplitude of the elastic pressure waves, which varies from one material to another also in terms of order of magnitude, is very weak and up to 10^{-6} times lower than atmospheric pressure. As a result, the electric signal from the transducer requires very high amplification (10^4 or 10^5 times) before it can be correctly elaborated (Fig. 1).

The signal picked up by a transducer is preamplified and transformed into electric voltage; it is then filtered to eliminate unwanted frequencies, such as the vibration arising from the mechanical instrumentation, which is generally lower than 100 kHz. Up to this point the signal can be represented as a damped oscillation. The signal is therefore analysed by a measuring system counting the emissions that exceed a certain voltage threshold (measured in volts (V)). This method of analysis is called Ring-Down Counting and is broadly used in the AE technique for the identification of defects (Fig. 1). As a first approximation, the number of counts N can be compared with the quantity of energy released during the loading process and we may assume that the corresponding increments grow proportionally to the extension of the crack faces. This technique also considers other procedures. For instance, by keeping track of the characteristics of the transducer used, and particularly of its damping, it is possible to consider all the

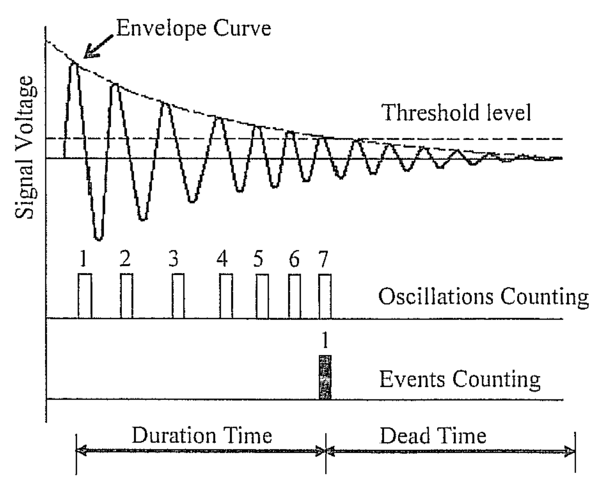

Fig. 1. Signal identified by the transducer and counting methods in AE technique.

oscillations produced by a single AE signal as a unique event and to replace the Ring-Down Counting with the Counting of Events method [12–14].

3. Time effects during damage evolution: A fractal theory

Each acoustic emission event, due to crack and elastic wave propagations in the damaging solids, has a characteristic duration τ that, according to the experimental evidence on earthquakes [10,11], we assume as a fractal (i.e., self-similar). Accordingly, distribution of the event durations, must be the following power law:

$$P(<\tau) = \frac{N(<\tau)}{N_{\max}} = 1 - \left(\frac{\tau_{\min}}{\tau}\right)^{D_T}, \tag{1}$$

where $N(<\tau)$ is the number of events with duration smaller than τ, $N_{\max}$ is the total number of events, $\tau_{\min}$ ($\ll \tau_{\max}$) is the minimum duration, and D_T (>0) is the fractal dimension of the self-similar time distribution of events.

The probability density function $p(\tau)$ is provided by derivation of the cumulative distribution function (1):

$$p(\tau) = D_T \frac{\tau_{\min}^{D_T}}{\tau^{D_T+1}}. \tag{2}$$

During fragmentation, the energy dissipation W in a volume V is [6]:

$$W \propto V^{D_S/3}, \tag{3}$$

where D_S (comprised between (2) and (3)) is the fractal dimension of the self-similar space distribution of fragments (assumed to follow the fractal size distribution of Eq. (2), replacing the duration of the event with the size of the fragment). Accordingly, the infinitesimal energy $\mathrm{d}W$ dissipated during a single event will follow Eq. (3), in which V represents the volume involved by the associated wave propagation. For isotropic three-dimensional wave propagation, along each axis the characteristic length of the event is $c\tau$, with c sound speed, so that $V \propto \tau^3$. Thus locally:

$$\mathrm{d}W \propto \tau^{D_S}\,\mathrm{d}N. \tag{4}$$

For two- or one-dimensional objects having characteristic size A or L respectively, the result is the same, since instead of Eq. (3) we have $W \propto A^{D_S/2}$ or $W \propto L^{D_S}$ with $A \propto \tau^2$ or $L \propto \tau$ and thus again Eq. (4).

The total energy dissipated will be consequently:

$$\begin{aligned} W &\propto \int_{\tau_{\min}}^{\tau_{\max}} \tau^{D_S}\,\mathrm{d}N = \int_{\tau_{\min}}^{\tau_{\max}} N_{\max}\tau^{P_S} p(\tau)\,\mathrm{d}\tau \\ &\propto N_{\max}\frac{D_T}{D_S - D_T}\tau_{\min}^{D_T}\left(\tau_{\max}^{D_S-D_T} - \tau_{\min}^{D_S-D_T}\right) \end{aligned}$$

$$\cong \begin{cases} N_{\max}\dfrac{D_T}{D_S - D_T}\tau_{\min}^{D_T}\tau_{\max}^{D_S - D_T}, & D_T < D_S, \\ N_{\max}\dfrac{D_T}{D_T - D_S}\tau_{\min}^{D_S}, & D_T > D_S. \end{cases} \tag{5}$$

On the other hand, the total (monitoring) time, statistically proportional to the total time duration, is given by:

$$t \propto \int_{\tau_{\min}}^{\tau_{\max}} \tau \, \mathrm{d}N = \int_{\tau_{\min}}^{\tau_{\max}} N_{\max}\tau p(\tau)\,\mathrm{d}\tau$$

$$\propto N_{\max}\frac{D_T}{1 - D_T}\tau_{\min}^{D_T}\left(\tau_{\max}^{1-D_T} - \tau_{\min}^{1-D_T}\right)$$

$$\cong \begin{cases} N_{\max}\dfrac{D_T}{1 - D_T}\tau_{\min}^{D_T}\tau_{\max}^{1-D_T}, & D_T < 1, \\ N_{\max}\dfrac{D_T}{D_T - 1}\tau_{\min}, & D_T > 1. \end{cases} \tag{6}$$

Here, according to the experimental acoustic emission monitoring, the events are assumed to be in series rather than in parallel. On the other hand, since a symbol of proportionality and not of equality is required in Eq. (6) for the definition of the monitoring time t, parallel events would be in principle allowed.

In addition, let us assume a duration "quantum" of size $\tau_{\min}$ = constant and make a statistical hypothesis of self-similarity, i.e., $\tau_{\max} \propto t$ (the larger the monitoring time, the larger the largest event). Accordingly, eliminating $N_{\max}$ from Eqs (5) and (6) we have:

$$\text{if } D_S \geqslant 1, \quad W \propto \begin{cases} t^{D_S}, & D_T < 1, \\ t^{1+D_S-D_T}, & 1 \leqslant D_T \leqslant D_S, \\ t, & D_T > D_S, \end{cases} \tag{7a}$$

$$\text{if } D_S < 1, \quad W \propto \begin{cases} t^{D_S}, & D_T < D_S, \\ t^{D_T}, & D_S \leqslant D_T \leqslant 1, \\ t, & D_T > 1. \end{cases} \tag{7b}$$

We have found that, the same time-scaling hold for the standard deviation σ_W of the energy if we formally replace in Eqs (7) W with σ_W and D_S with $2D_S$. A similar fractal approach on size-scaling rather than on time has already been proposed for predicting the size-effects on the mean values and on the standard deviations for the main mechanical properties of materials [7], starting from the space-scaling of the energy [6].

Note that usually $\mathrm{D} - 1 < D_S < \mathrm{D}$ with $\mathrm{D} = 1, 2, 3$ object dimension [6]. From Eqs (7), $W \propto t^{\beta_t}$ with $1 \leqslant \beta_t \leqslant D_S$ if $D_S \geqslant 1$ or $D_S \leqslant \beta_t \leqslant 1$ if $D_S < 1$, i.e., in general:

$$W \propto t^{\beta_t}, \quad 0 \leqslant \beta_t \leqslant 3. \tag{8}$$

The corresponding fractal size-scaling on acoustic emission during cracking of solids has already been proposed by the same authors [8,9], on the basis of the fractal fragmentation law [6].

The experimental validation of the time-scaling of Eq. (8) represents the aim of the next sections.

4. Damage detection in concrete structures

The AE method, which is called Ring-Down Counting or Event-Counting, considers the number of waves beyond a certain threshold level (measured in Volt) and is widely used for defect analysis [12–14]. As a first approximation, in fact, the cumulative number of counts N can be compared with the amount of energy released during the loading process, assuming that both quantities increase with the extent of damage (i.e., $W \propto N$, and the energy must be additive). By means of this technique, we have analysed the evolution of cracks and estimated the released strain energy during their propagation in structural members. In particular, the damage evolution in several structures by an extensive experimental analysis in space and time have been investigated. Among these structures we also analysed the damage evolution in two pilasters (Fig. 2) sustaining a viaduct along an Italian highway built in the 1950s [8,9]. The monitoring process consists of recording the AE signals arising from the cracking phenomena occurring in two piers, referred to as P1 and P2. Pier P1 was monitored for ca 4126 hours. Figure 3(a) shows a chart of the differential counts of the oscillations recorded per hour, and a chart of the cumulative counts obtained from the sum of the latter. From the cumulative count chart it may therefore be seen that pier behaviour is not stable, since damage evolves progressively over time. The differential count chart displays, on an expanded scale, the predominant events, i.e., the emission peaks characterising the critical moments in crack propagation. These events occur at intervals of hundreds of hours. Their occurrence shows that damage is not due to traffic-induced vibrations, whose frequent oscillations remain within the structure's elasticity limits, and it is caused instead by other slow-rate phenomena, such as foundation

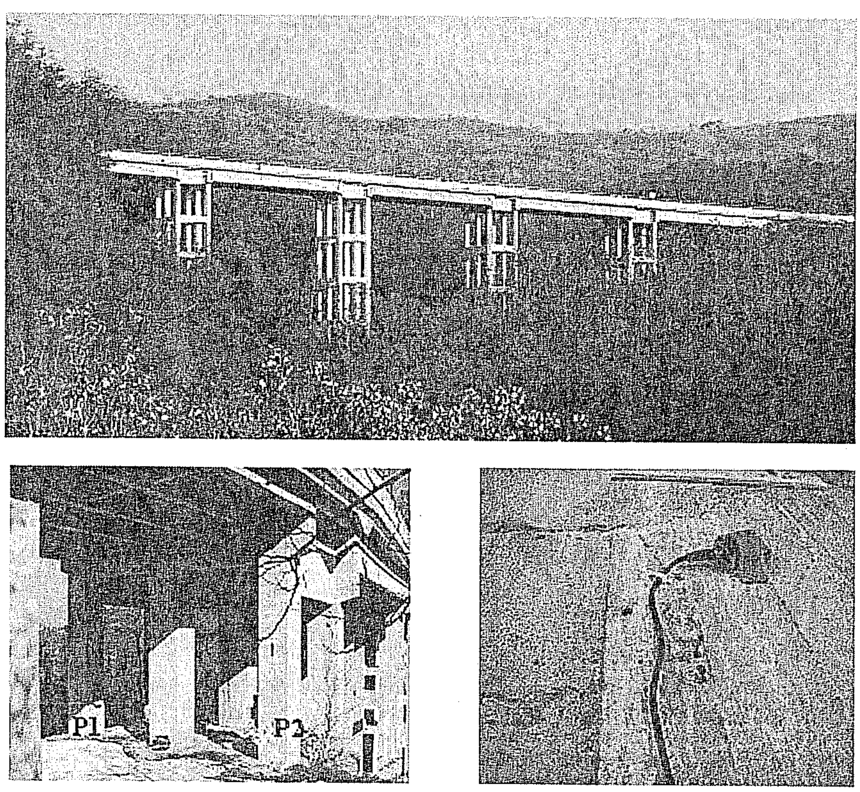

Fig. 2. The viaduct and the monitored pilasters P_1 and P_2 with the applied AE sensor.

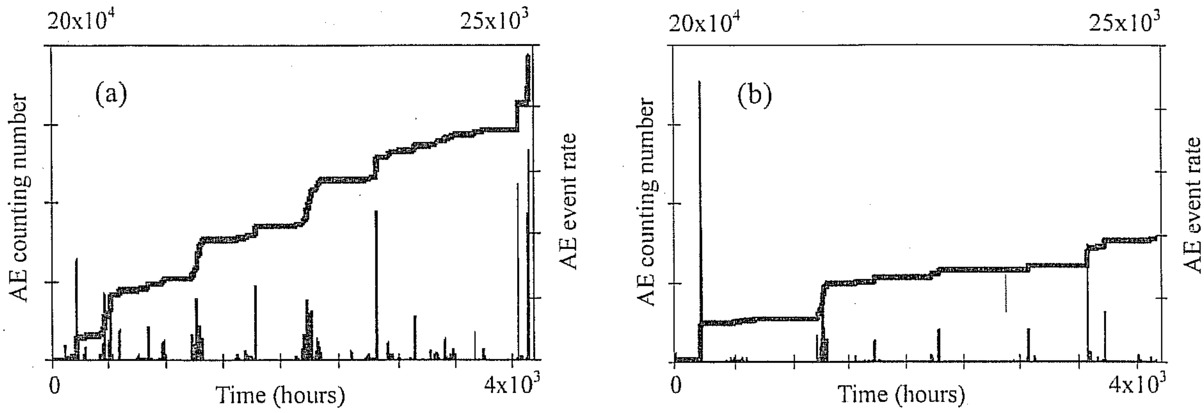

Fig. 3. Piers P1 (a), P2 (b) AE monitoring data.

settlements due to soil creep. Pier P2 was also monitored for 4126 hours running. From the cumulative count chart of Fig. 3(b) we find that the total number of oscillations for this pier is ca 40% the number recorded for pier P1. Accordingly, damage in P2 is characterised by a slower evolution compared to P1.

5. Acoustic emission monitoring: Experimental evidence

From the monitoring pilasters we drilled some concrete cylindrical specimens in order to detect the mechanical properties of the material under compression and to evaluate the scale-effects on AE activity in size [8] and time. For each pilaster, three different specimen diameters d are considered in a maximum scale range of 1 : 3.4. The specimens present three different slendernesses: $\lambda = h/d = 0.5$, 1.0 and 2.0, with d chosen equal to 27.7, 59, 94 mm, respectively. For each of these nine geometries, three specimens have been tested, for a total of 54 cases (two pilasters). All compression tests were performed under displacement control, by imposing a constant rate to the upper loading platen. We adopted a displacement rate equal to 10^{-4} mm/s for all specimens, in order to obtain a very slow-crack growth and to detect all possible AE signals. In this way, we were able to capture also the softening branch of the stress-strain diagrams. The experimental results are summarized in Table 1. For details on the concrete specimens, machine and test conditions the reader should refer to [8].

According to Eq. (8) and to $W \propto N$, an energy damage parameter η during the specimen testing, can be defined as:

$$\eta \equiv \frac{W}{W_{\max}} = \frac{N}{N_{\max}} = \left(\frac{t}{t_{\max}}\right)^{\beta_t}, \tag{9}$$

where "max" refers to the reaching of the maximum stress (that we chose as the critical condition). From Eq. (9) the experimental values of β_t, describing the time-scaling of the energy dissipated or released can be deduced (according to the fractal theory, it is expected not to be strongly dependent on test conditions).

An example of experimental space–time effects on the tested specimens are given in Fig. 4. After an initial transient period ($0 < t/t_{\max} < 0.4$) [15], a true power-law for the time-scaling is observed. From the best-fitting in the bilogarithmic plane (Fig. 4), for the tested specimen ($d = 59$ mm, $\lambda = 1$) we

Table 1
Average values for the specimens obtained form pilasters P1 and P2

Specimen number	Diameter d [mm]	Slenderness $\lambda = h/d$	Pilaster P1			Pilaster P2		
			Peak stress σ_u [MPa]	N_{max} at σ_u	β_t	Peak stress σ_u [MPa]	N_{max} at σ_u	β_t
1	27.7	0.5	91.9	1186	1.40	84.7	1180	1.38
2	27.7	1.0	62.8	1191	1.41	46.7	1181	1.46
3	27.7	2.0	48.1	1188	1.48	45.8	1186	1.67
4	59.0	0.5	68.1	8936	2.12	57.5	8924	2.39
5	59.0	1.0	53.1	8934	1.49	41.7	8930	2.52
6	59.0	2.0	47.8	8903	2.30	38.2	8889	2.41
7	94.0	0.5	61.3	28502	2.90	45.2	28484	2.84
8	94.0	1.0	47.8	28721	2.09	38.2	28715	2.21
9	94.0	2.0	44.1	28965	2.80	38.1	28956	2.92

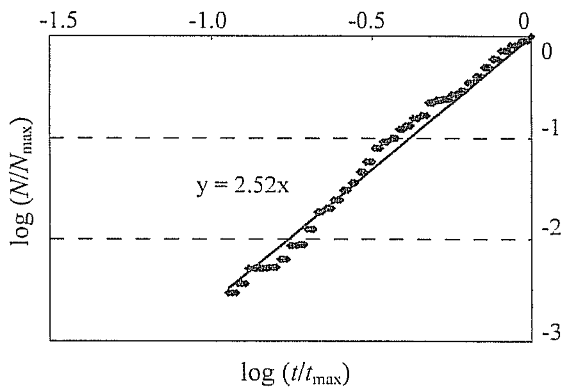

Fig. 4. Time scaling in damage evolution.

obtain the slope $\beta_t = 2.52$. Similar results can be observed for stress and strain dependencies. The size-scaling on N_{max} is also represented as a function of the volume specimen (Fig. 5), fitted to experimental data. A slope in the log–log plane between 2/3 and 1 (experimentally close to 0.77) emphasizes that the energy dissipation occurs in a fractal domain, intermediate between a surface and a volume (for details see [6,8]). The β_t values plotted versus the specimen diameters are reported in Fig. 6. The observed trend is an increase of the β_t values by increasing the specimen diameter (Table 1). The experimental time-scaling agree with the fractal law of Eq. (8), giving exponent in the range $(0, 3)$.

The damage level of the monitored structure can be estimated on the basis of AE data obtained from the tested specimens. During the observation period (172 days), we obtained a number of events $N \cong 2 \times 10^5$ for the more damaged pilaster P_1, and $N \cong 8 \times 10^4$ for the less damaged P_2, respectively (Fig. 3). Since the volume of each pilaster is about 2×10^6 cm^3, extrapolating from Fig. 5, we estimate the critical number of AE for the pilasters equal to $N_{max} \cong 11.51 \times 10^6$. Inserting the values of N and N_{max} into Eq. (9), and assuming an exponent $\beta_t = 2.52$ (a more conservative choice would be 3), we obtain $t/t_{max} \cong 0.2$ for pilaster P_1, and $t/t_{max} \cong 0.14$ for pilaster P_2. The lifetime of these structural elements is therefore estimated, corresponding to the achievement of the maximum number of events, at respectively 2.4 and 3.4 years.

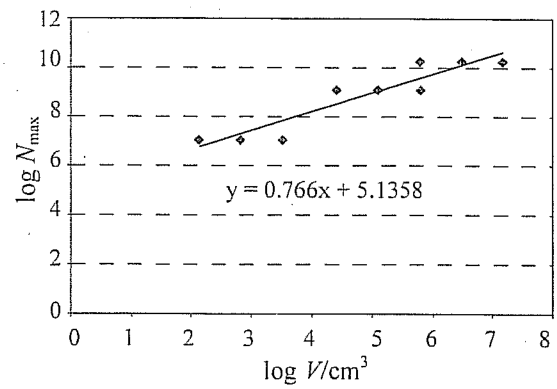

Fig. 5. Space scaling in damage evolution.

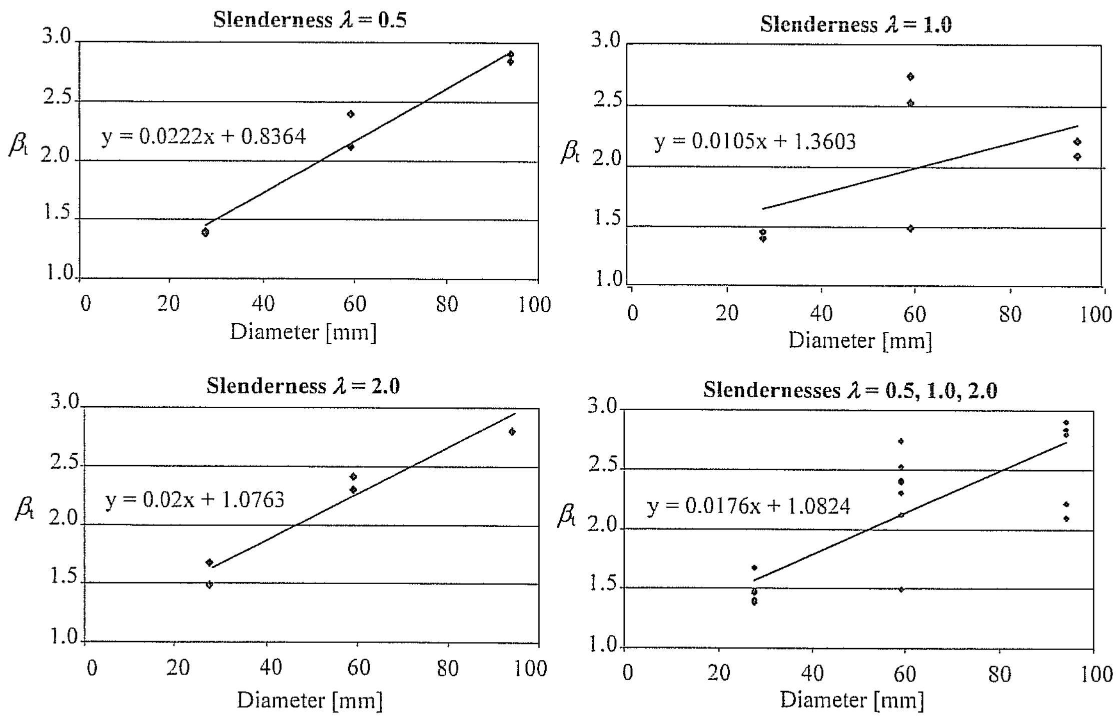

Fig. 6. Life-time exponent β_t plotted vs. specimens diameter.

6. Conclusions

In this paper a fractal theory for predicting the time-scaling of the damage evolution in cracking solids has been presented and investigated experimentally by acoustic emission technique. The analytical result, summarized in Eq. (8), seems to be confirmed by the experimental evidence on acoustic emission, showing power law damage evolution with fractal exponents β_t comprised between 0 and 3. Coupling space–time effects, the life time predictions for structures can be estimated in exercise conditions and *in situ*.

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