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Mechanical properties of the hierarchical honeycombs with stochastic Voronoi sub-structures

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Abstract – The introduction of hierarchy into structures has been credited with changing mechanical properties. In this study, periodically hierarchical honeycomb with irregular sub-structure cells has been designed based on the Voronoi tessellation algorithm. Numerical investigation has been performed to determine the influence of structural hierarchy and irregularity on the in-plane elastic properties. Irregular hierarchical honeycombs can be up to 3 times stiffer than regular hexagonal honeycombs on an equal density basis. Both the stiffness and Poisson's ratio of the hierarchical honeycomb are insensitive to the degree of regularity, and depend on the cell-wall thickness-to-length ratio of the super-structure. Increasing the relative lengths of the super- and sub-structures results in the increment of Young's modulus, whereas Poisson's ratio almost remains constant varying from 1.0 to 0.7.

Introduction. – Due to their low weight and specific mechanical properties in the aspects of stiffness, strength and toughness, etc., the regular hexagonal honeycombs have many potential multifunctional applications in the fields of mechanical engineering, aerospace engineering, civil engineering and so on. They are used as the core of the sandwich panels and generally there are two kinds of topologies: one is the in-plane sandwich panels with the honeycomb columns parallel to the top and bottom surface panels and the other one is the out-of-plane sandwich panels with the honeycomb columns perpendicular to the top and bottom surface panels.

With respect to applications in the form of in-plane sandwich panels, the paper honeycombs were used to fabricate the structural and cushioning components [1]; the metal honeycombs were also good candidates both for the heat sink for electronic devices [2,3] and for the protective layer to protect structures from the high intensity dynamic loads by mitigating blast waves [4]; moreover, the pressure adaptive honeycomb can achieve a novel concept for smart morphing aircraft structures [5,6]. Regarding to the application of the out-of-plane sandwich panels, the superalloy honeycomb sandwich thermal protection systems attained good heat shielding characteristics and thermostructural performances for hypersonic flight vehicles [7]; the metal honeycombs had the abilities to tailor impact resistance for racing cars [8], low floor bus [9], train application [10], etc.

To explore the multifunctional properties of the regular hexagonal honeycombs more efficiently, mimicking the hierarchical structures widely existing in Nature and man-made materials, many different kinds of hierarchical honeycombs are proposed [11–29]. Taylor *et al.* [30] proposed the functionally graded hierarchical honeycombs and showed that the increase of the elastic modulus could reach up to 75%. The research group of Vaziri proposed

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the self-similar isotropic [31] and anisotropic [32] hierarchical honeycombs by replacing each three-edge vertex of a base hexagonal network with a similar but smaller hexagon of the same orientation. The isotropic hierarchical honeycombs of first and second order can be up to 2.0 and 3.5 times stiffer [31] and the anisotropic hierarchical honeycombs of first to fourth order can be 2.0–8.0 times stiffer and at the same time up to 2.0 times stronger than their corresponding regular honeycombs at the same mass [32]. To greatly (more than 10 times) increase the in-plane stiffness of the hexagonal honeycombs at a wide density range, the isotropic [27,33] and anisotropic [34] multifunctional hierarchical honeycombs (MHH) are constructed by replacing the cell wall of the original regular hexagonal honeycombs (ORHH) with the equal-mass isotropic sub-structures, including triangular, Kagome or chiral honeycombs. By replacing the cell wall of the ORHH with sandwiched rectangle struts core and with sandwiched triangular struts core, Xu *et al.* [35] proposed the second-order hierarchical hexagonal lattice structures and showed that the structure hierarchy is favorable for the periodic lattice structure to filtering or guiding wave at some circumstances to meet the demands of engineering.

Sub-structures of the above-mentioned hierarchical honeycombs are mainly regular honeycomb lattice, and relatively little attention has been paid for the stochastic Voronoi honeycombs. Therefore, in this letter, the hierarchical honeycombs with stochastic Voronoi honeycomb sub-structures are proposed by replacing the solid cell wall of the ORHH with the equal mass. Effective elastic modulus and Poisson's ratio of this new kind of hierarchical honeycombs are numerically studied. Except the improvement on the in-plane stiffness, the stochastic Voronoi honeycomb sub-structures could also have better structural-acoustic properties than the regular honeycomb ones [36].

Geometrical description. – As already mentioned above, hierarchical honeycombs with stochastic Voronoi honeycomb sub-structures were constructed by replacing the solid cell walls of the ORHH with stochastic Voronoi honeycomb sub-structures. The representative unit cells are shown in fig. 1. Moreover, the hierarchical honeycomb is fully periodic. Several parameters that describe the nature of hierarchy in honeycombs are introduced in the following sections.

The irregular Voronoi honeycomb mimics the nucleation and growth of bubbles under these two assumptions: a) the bubbles all nucleate simultaneously in a determinate region of space; b) they grow at the same linear rate in all directions [37]. Various methods have been reported for the generation of nucleation points in the literature. In this work, a method proposed by Zhu *et al.* [38] was utilized to construct random irregular Voronoi honeycombs. The nucleation points were randomly generated one by one, and ensured the distance between any two points to be larger than a minimum δ. The degree of the cell

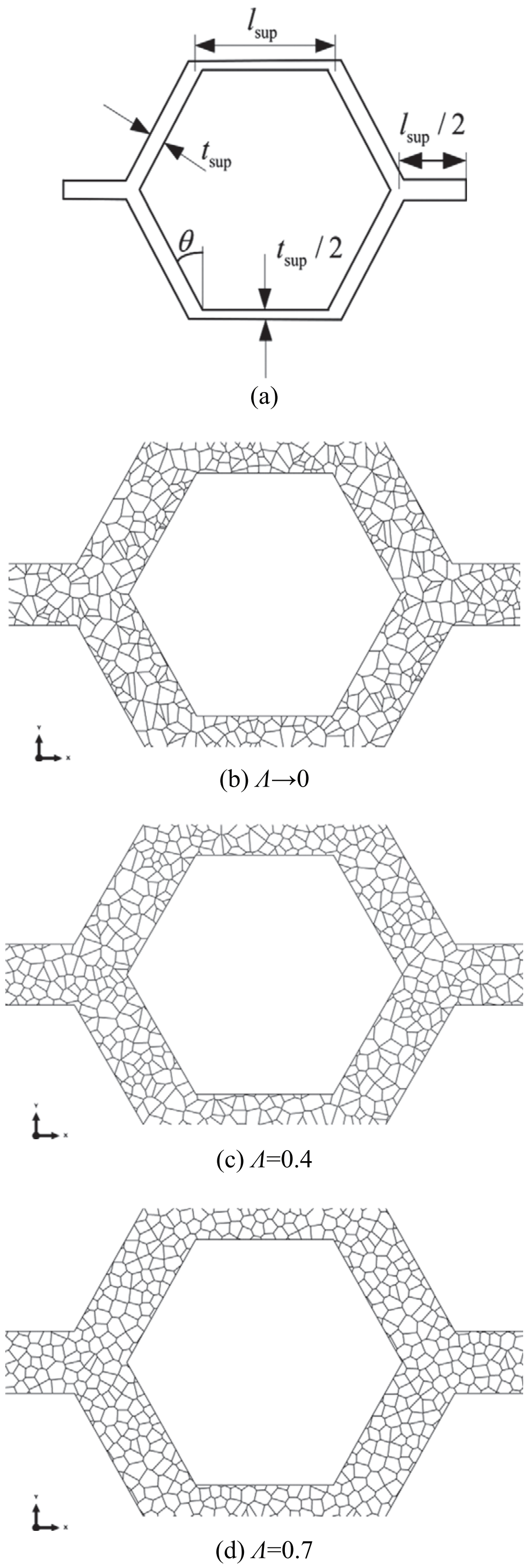

Fig. 1: The cell geometry and parameters for a conventional hexagonal honeycomb unit cell (a); the periodic unit cell of hierarchical honeycomb with different degree of regularity: (b) $\Lambda \to 0$; (c) $\Lambda = 0.4$; (d) $\Lambda = 0.7$.

regularity Λ is thus defined as

$$\Lambda = \frac{\delta}{d_0}, \tag{1}$$

where $d_0 = \sqrt{\frac{2A_0}{N\sqrt{3}}}$ is the distance between two neighbouring cells in a perfect regular hexagonal honeycomb with the same number of complete cells; A_0 is the control area; N is the total number of complete cells. $\Lambda \to 1$ implies a perfect regular hexagonal honeycomb, whereas $\Lambda \to 0$ represents a completely random Voronoi honeycomb as shown in fig. 1. The relative density, $\bar{\rho}$, is defined as the ratio of the macroscopic density of cellular structure to the density of cell wall material. With the assumption that all cell walls have the same thickness t, the relative density is determined in a normalized form as

$$\overline{\rho} = \frac{t\sum_{i=1}^{N} l_i}{A_0}, \tag{2}$$

where l_{i} are the cell wall lengths, N is the total number of cell wall. The cell-wall thickness-to-length ratio of the super-structure, α, is defined as its thickness t_{sup} divided by its length l_{sup} (subscripts "sup" and "sub" represent super- and sub-structure, respectively) as shown in fig. 1:

$$\alpha = \frac{t_{\mathrm{sup}}}{l_{\mathrm{sup}}}. \tag{3}$$

The ratio of the lengths in the super- and sub-structure is the hierarchical length ratio λ and is defined as

$$\lambda = \frac{l_{\mathrm{sup}}}{\widehat{l}_{\mathrm{sub}}}, \tag{4}$$

where $\widehat{l}_{\mathrm{sub}}$ is the average wall length of the sub-structure. It can effectively increase the number of sub-cells spanning the length of the super-structure via changing the parameter λ.

Numerical model. – The effect of irregularity on in-plane elastic properties of the periodic hierarchical honeycomb was determined using the software ABAQUS/standard. Two-dimensional finite-element models were generated for each representative unit cell using 2D Timoshenko beam elements (B21). The beam thickness can be determined by eq. (2) for the designated values. The material model was linear elastic and small strains with a Poisson's ratio 0.3. Periodic boundary conditions were imposed along the opposite boundary of the representative unit cell [39]. Thus, the translation displacements u and rotation ω of counterpart nodes of the boundaries are constrained to each other as follows:

$$u_i^p - u_i^q = \varepsilon_{ij}\left(x_j^q - x_j^p\right), \qquad \omega^p - \omega^q = 0, \tag{5}$$

where $i,\ j = 1,2$ denote degrees of freedom in the two-dimension problem and p, q refer to the nodes on opposite sides of the unit cell; ε_{ij} is the average macroscopic strain; x is the coordinate of the node.

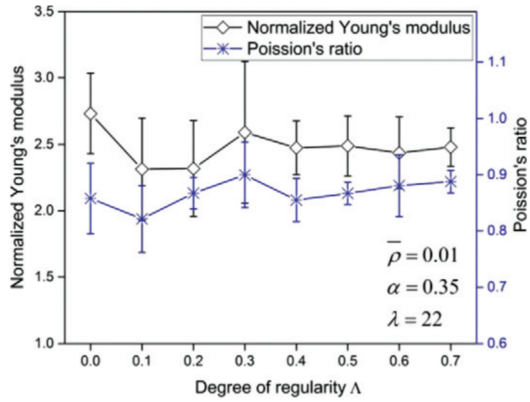

Fig. 2: (Colour on-line) Effect of cell regularity on the elastic properties.

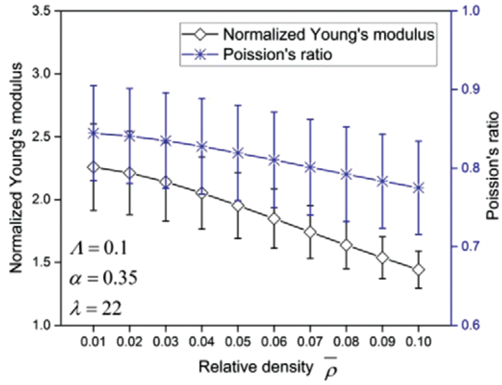

Fig. 3: (Colour on-line) Effect of relative density on the elastic properties.

Results and discussion. –

The effect of regularity. In this section the influence of regularity on the elastic properties of periodic hierarchical honeycombs is presented. Analyses were performed on unit cells over a wide range of regularities as shown in fig. 2, where the Young moduli of the hierarchical honeycombs were normalized by that of a regular hexagonal honeycomb at the same mass. It is shown in fig. 2 that cell irregularity does not appear to have a significant effect on the Young moduli, leading only to a slightly increment as $\Lambda \to 0$, whereas the Young moduli of the random Voronoi honeycomb without hierarchy increase significantly as $\Lambda \to 0$ [38]. However, compared with their regular counterpart, the introduction of the irregular substructure resulted in an increase in stiffness at least by 150%. In contrast, the Poisson ratios almost remain constant regardless of the cell regularity.

The effect of relative density. Figure 3 presents the non-dimensional Young moduli and the Poisson ratios as functions of relative density with the fixed parameters $\Lambda = 0.1$, $\alpha = 0.35$ and $\lambda = 22$. Both the normalized Young moduli and Poisson's ratios decrease gradually with increasing relative density $\bar{\rho}$. The present FE results for hierarchical structures are similar to the estimate by

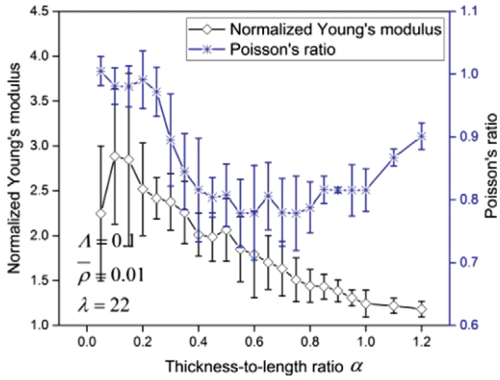

Fig. 4: (Colour on-line) Effect of thickness-to-length ratio on the elastic properties.

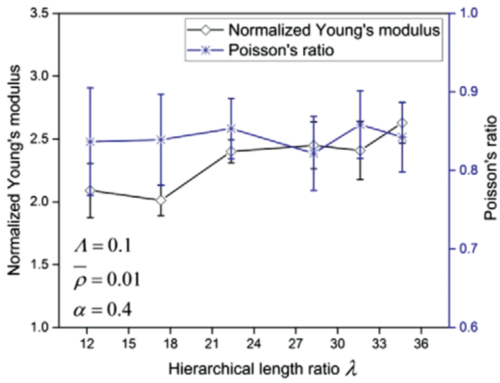

Fig. 5: (Colour on-line) Effect of hierarchical length ratio on the elastic properties.

Zhu *et al.* [38] for the random Voronoi honeycomb without hierarchy. Compared to the cell regularity, the relative density effect is more pronounced regardless of hierarchy.

The effect of thickness-to-length ratio. The effects of the cell-wall thickness-to-length ratio α of the hierarchical honeycombs on the non-dimensional effective Young moduli and Poisson's ratio are examined in this section. The hierarchical honeycombs with stochastic Voronoi substructures degenerate to be a random Voronoi honeycomb gradually when α approaches the upper bound $\sqrt{3}$ [33]. Therefore, we maintained $\Lambda = 0.1$ $\bar{\rho} = 0.35$ and $\lambda = 22$ with α changing from 0.05 to 1.2. It is shown in fig. 4 that the Young modulus is very sensitive to α. The normalized Young moduli increase first, and then decrease with the increase of α. There exists a maximum Young modulus which could be 300% larger than that of a first-order regular hexagonal honeycomb. Therefore, the optimal effective Young modulus may exist for each unique Voronoi tessellation. The Poisson ratio decreases with increasing α until the lowest value 0.7 ($\alpha = 0.6$), and then this trend is reversed as $\alpha > 1$.

The effect of Hierarchical length ratio. To investigate the effects of hierarchical length ratio λ on the elastic properties, we considered the fixed constant parameters $\Lambda = 0.5, \bar{\rho} = 0.01$ and $\alpha = 0.4$ as a case study. The non-dimensional Young moduli and the Poisson ratio are plotted in fig. 5. A significant increment in the average effective Young moduli can be found when λ increases from 12.2 to 34.6, whereas no obvious variation in the Poisson's ratio can be identified. It is found that the smaller the sub-cell size is, the larger the stiffness can be.

Summary. – In the letter, we proposed a new hierarchical honeycomb with irregular sub-structure and discussed the possibility of designing a new class of hierarchical honeycomb with the tailorable properties. The results showed that the effective Young modulus of the hierarchical honeycomb could be 300 percent larger than these of first-order regular hexagonal honeycomb at the same mass. In contrast, there was no obvious indication that the effective Young modulus and the Poisson ratio were affected significantly by the irregularities. Both the Young modulus and the Poisson ratio decreased gradually with increasing the relative density. In addition, the cell-wall thickness-to-length ratio of the super-structure had noticeable effect on the Young modulus and the Poisson ratio. Although the effective Young modulus increased as the hierarchical length ratio increased, the Poisson ratio was relatively insensitive to the hierarchical length ratio. Based on this work, it is suggested to further optimize the hierarchical structure in terms of both the cell wall misalignment and the sub-structures (substituting the hexagon with the triangular or Kagome as proposed in ref. [33]).

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REFERENCES

[1] Lu L. X., Sun Y. P. and Wang Z. W., *Packag. Technol. Sci.*, **18** (2005) 141.
[2] Lu T. J., *Int. J. Heat Mass Transfer*, **42** (1998) 2031.
[3] Gu S., Lu T. J. and Evans A. G., *Int. J. Heat Mass Transfer*, **44** (2001) 2163.
[4] Wadley H. N. G., *Philos. Trans. R. Soc. London, Ser. A*, **364** (2006) 31.

[5] Vos R. and Barrett R., *Proc. SPIE*, **7647** (2010) 76472B.
[6] Vos R., Barrett R. and Romkes A., *J. Intell. Mater. Syst. Struct.*, **22** (2011) 1041.
[7] Ko W. L., *Heat Shielding Characteristics and Thermostructural Performance of a Superalloy Honeycomb Sandwich Thermal Protection System (TPS)*, NASA/TP-2004-212024.
[8] Thompson R. W., Matthews F. L. and O'Rourke B. P., *Mater. Des.*, **16** (1995) 131.
[9] Shin K. B., Lee J. Y. and Cho S. H., *Compos. Struct.*, **84** (2008) 228.
[10] Zinno A., Fusco E., Prota A. and Manfredi G., *Compos. Struct.*, **92** (2010) 2208.
[11] Lakes R., *Nature*, **361** (1993) 511.
[12] Gibson L. J. and Ashby M. F., *Cellular solids: Structure and Properties*, second edition (Cambridge University Press, Cambridge) 1997.
[13] Pugno N. M., *Nanotechnology*, **17** (2006) 5480.
[14] Gao H., *Int. J. Fract.*, **138** (2006) 101.
[15] Pugno N. and Carpinteri A., *Philos. Mag. Lett.*, **88** (2008) 397.
[16] Carpinteri A. and Pugno N. M., *Int. J. Fract.*, **150** (2008) 221.
[17] Qin Z., Kreplak L. and Buehler M. J., *PLoS ONE*, **4** (2009) e7294.
[18] Fratzl P. and Weinkamer R., *Prog. Mater. Sci.*, **52** (2007) 1263.
[19] Pugno N. M. and Chen Q., *Eng. Proc. Phys. Eng.*, **10** (2011) 3026.
[20] Chen Q. and Pugno N. M., *Compos.: Part B*, **42** (2011) 2030.
[21] Chen Q. and Pugno N. M., *EPL*, **97** (2012) 26002.
[22] Chen Q. and Pugno N., *Eur. J. Mech. A/Solids*, **34** (2012) 120.
[23] Chen Q. and Pugno N. M., *EPL*, **98** (2012) 16005.
[24] Chen Q. and Huang S., *Mater. Lett.*, **95** (2013) 89.
[25] Chen Q. and Pugno N. M., *J. Mech. Behav. Biomed. Mater.*, **19** (2013) 3.
[26] Pugno N. and Sun Y., *Materials*, **6** (2013) 699.
[27] Sun Y. and Pugno N. M., *Compos. Struct.*, **106** (2013) 681.
[28] Vigliotti A. and Pasini D., *Mech. Mater.*, **62** (2013) 32.
[29] Banerjee S., *Mech. Mater.*, **72** (2014) 19.
[30] Taylor C. M., Smith C. W., Miller W. and Evans K. E., *Int. J. Solids Struct.*, **48** (2011) 1330.
[31] Ajdari A., Jahromi B. H., Papadopoulos J., Nayeb-Hashemi H. and Vaziri A., *Int. J. Solids Struct.*, **49** (2012) 1413.
[32] Oftadeh R., Haghpanah B., Papadopoulos J., Hamouda A. M. S., Nayeb-Hashemi H. and Vaziri A., *Int. J. Mech. Sci.*, **81** (2014) 126.
[33] Sun Y., Chen Q. and Pugno N., *Compos. Struct.*, **107** (2014) 698.
[34] Sun Y., Wang B., Pugno N., Wang B. and Ding Q., *Compos. Struct.*, **131** (2015) 616.
[35] Xu Y. L., Chen C. Q. and Tian X. G., *J. Vib. Acoust.*, **136** (2013) 122.
[36] Denli H. and Sun J. Q., *J. Sound Vib.*, **301** (2007) 93.
[37] Silva M. J., Hayes W. C. and Gibson L. J., *Int. J. Mech. Sci.*, **37** (1995) 1161.
[38] Zhu H. X., Hobdell J. R. and Windle A. H., *J. Mech. Phys. Solids*, **49** (2001) 857.
[39] Chen C., Lu T. J. and Fleck N. A., *J. Mech. Phys. Solids*, **47** (1999) 2235.