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Fractal coupled theory of drilling and wear

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Abstract. Drilling perforations and tool wear are intimately and mutually connected by fracture propagations at different size-scales. To study this interaction phenomenon, we propose an *ad hoc* developed fractal coupled theory. Describing the two processes in terms of drilling and wear velocities, the theory is able to predict the relation between these two quantities. The result is represented by a power law between wear and drilling velocities with exponent comprised between 2/3 and 3/2. Some experimental tests on different materials like mortar, concrete and reinforced concrete have also been performed. Theoretical predictions and experimental results agree satisfactorily.

Key words: Drilling, fractal, fracture, multiscale, wear.

1. Introduction

Drilling and wear are different forms of the same physical phenomenon, i.e., fracture. Drilling (Carpinteri and Pugno, 2002a) can be considered simply artificial fragmentation. On the other hand, wear is studied in the Tribology science. The word Tribology is derived from the greek word *tribos,* meaning "rubbing" and it is defined as "the science and technology of interacting surfaces in relative motion and of the practices related thereto". It embraces the scientific investigation of all types of friction, lubrication and wear and also the technical application of tribological knowledge (Rabinowicz, 1995).

Focusing our attention on wear, we can distinguish four main forms of wear (Rabinowicz, 1995):

1. *Adhesive wear* occurs when two smooth bodies are sliding one over each other, and fragments are pulled off one surface and adhere to the other. It derives from the strong adhesive forces set up whenever atoms come into intimate contact.
2. *Abrasive wear* occurs when a rough hard surface, or a soft surface containing hard particles, slides on a softer surface and ploughs a series of grooves on it.
3. *Corrosive wear* occurs when sliding takes place in a corrosive environment. In the absence of sliding, the products of the corrosion will form a film on the surfaces. This film tends to slow down or even arrest the corrosion. However, the sliding action wears the film away, so that the corrosive attack continues.
4. *Surface fatigue wear* occurs during repeated sliding or rolling over a track. The repeated loading and unloading cycles to which the materials are exposed may

induce the formation of surface or subsurface cracks, which eventually will result in the formation of large fragments, leaving large pits in the surface.

Other forms of wear are

5. *Fretting* occurs when contacting surfaces undergo oscillatory tangential displacement of small amplitude.
6. *Erosion* is a process in which a particle carried in a fluid medium hits a solid surface and removes material from it (low-speed, high-speed and cavitation erosion).
7. *Impact wear* happens when two surfaces collide while having large relative velocities normal to their interface.
8. *Brittle fracture wear* occurs during sliding in brittl materials, when a characteristic series of cracks is observed in the wear track. Subsequently, large wear particles tend to be produced during surface breakup.

The examination of a failed sliding member, to determine the type of wear responsible, can be a complex process; several pioneer manuscripts deal with this topic (Snook, 1953; Burwell, 1957; Furman et al., 1957; Love, 1957; Sprague and Dundy, 1959; Eyre, 1976).

In this paper we try to unify all these different wear mechanisms on the basis of their particle production, applying the universal law for the energy dissipation during fragmentation (Carpinteri and Pugno, 2002b). A fractal approach is expected to be powerful in the context of wear, as emphasized in a recent analysis on erosion due to space debris impacts (Carpinteri and Pugno, 2004a).

Considering tool wear during drilling perforations, the studies of impregnated diamond core-bits have been largely concentrated on the diamond wear. The documentation on metal matrix wear and the wear of the entire impregnated diamond tool is rare and substantially experimental (Miller and Ball, 1990, 1991; Tian and Tian, 1994). Experimental results on micro-bit drilling tests indicate that the penetration per revolution is one of the most important factors influencing the wear of impregnated diamond bits. In fact, the bit weight loss per distance drilled increases drastically with an increase in the penetration per revolution. On the other hand, the bit weight loss per distance drilled is found to decrease slightly with an increase in the rotational speed.

Referring to the described wear types, the wear modes for the drilling process are substantially of brittle fracture, abrasive, adhesive and erosion. Under ordinary drilling conditions, the brittle fracture wear is the predominant wear mode. This wear mechanism generates sharp diamonds. On the other hand, under very small applied thrust loads, the abrasive wear is the predominant wear mode. This wear mechanism generates excessive wear flats at the diamond cutting edges that often result in rapid decrease of the penetration rate. Furthermore, under very high penetration rates, the so-called “micro-burn” phenomenon takes place at the cutting surface. Drilling detritus particles adhere to the matrix between diamond grits. They are delaminated under rock abrasion, causing rapid adhesive wear of the impregnated diamond bit. In addition, the flow of drilling detritus constitutes the major abrasive third body against the matrix; in this case, the wear of the bit matrix is a mixed micro-ploughing process of erosion.

In order to achieve the proper wear rate of bit matrix in steady state drilling, it is important to maintain an optimal value of penetration per revolution to produce the right amount of drilling detritus under the bit cutting face and a proper diamond cracking for maintaining rock cutting ability. If the penetration per revolution is too large, excessive wear of diamonds and metal matrix takes place, shortening the working life of the diamond bits. On the other hand, if the penetration per revolution is too small, excessive wear flats of diamond cutting edges result in a rapid decrease of the penetration rate.

As previously argued, drilling perforation and tool wear appear complex phenomena, intimately and mutually connected. In the present paper, we propose a coupled theory, based on statistical and fractal concepts (Mandelbrot, 1982; Feder, 1988; Turcotte, 1992; Carpinteri and Pugno, 2002a,b), to describe the phenomena from a global point of view. According to Carpinteri and Pugno (2002a), the drilling process is described in terms of drilling velocity. A similar global parameter, the wear rate, is introduced to describe the tool wear. The coupled fractal theory is able to predict the relationship between these two quantities.

2. Coupled law of drilling and wear velocities

In this section we present a coupled law of drilling and wear velocities based on classical wear concepts. The wear loss w is defined as the volume removed V per unit area A and per unit length x of sliding (Horbogen, 1986), i.e.,

$$w = \frac{V}{xA} = \frac{\dot{V}}{\dot{x}A}, \tag{1}$$

where the dot over the symbol has the meaning of time derivative.

On the other hand, the wear coefficient k is defined as the probability of wear in the portion of the surface which is interacting (Horbogen, 1986)

$$w = k\frac{A_{\text{contact}}}{A} = k\frac{F}{AH}, \tag{2}$$

where A_{contact} is the portion of the nominal area A in contact, H is the hardness (of the worn material) and F is the thrust. The main difference between the wear loss w and the wear coefficient k is that the first is not a material property being thrust-dependent. On the other hand, the second, in the classical approach, can be considered as a material property and it is obviously thrust-independent.

Assuming that all the friction energy is dissipated in wear (e.g., a soft body sliding on a rigid surface), we have

$$\mathrm{d}W = \mu F \mathrm{d}x, \tag{3}$$

where μ is the friction coefficient (between the two materials in contact). Eliminating F from Eqs. (2) and (3), we obtain

$$w = \frac{k}{AH\mu}\frac{\mathrm{d}W}{\mathrm{d}x} = \frac{k}{AH\mu}\frac{\dot{W}}{\dot{x}}. \tag{4}$$

From Eq. (1) and from the definition of wear resistance $S = \dot{W}/\dot{V}$, ratio of dissipated power to removed volume per unit time (Paone and Bruce, 1963; Carpinteri and Pugno, 2002), we have

$$w = \frac{\dot{V}}{\dot{x}A} = \frac{1}{\dot{x}A}\frac{\dot{W}}{S}. \tag{5}$$

Combining Eqs. (4) and (5), we obtain the wear resistance S, which is a macroscopic parameter, as a function of microscopic material constants

$$S = \frac{\mu H}{k}. \tag{6}$$

The developed theory has permitted to obtain the relationship between the classical wear coefficient k and the wear strength S (of the tool with hardness H). If Eq. (6) were applied to the drilling process, S would represent the drilling strength (of the base-material). In this latter case, the drilling strength S is a function of the friction coefficient μ, of the hardness H and of the wear coefficient k of the base material.

It is important to emphasize that the derivation of Eq. (6) is rigorous only for a pure wear (or drilling) process, i.e., when we assume that the whole power is entirely dissipated in wear (or drilling). This can not be assumed (by definition) in a coupled (e.g., drilling-wear) theory, for which the Equation (6) must be modified taking into account that not the whole power is dissipated in wear (or drilling). According to these considerations, the hypothesis used in Eq. (3) must be replaced with the following relationship:

$$\mathrm{d}W_i = \alpha_i \mu F \mathrm{d}x, \quad \alpha_i = \frac{\dot{W}_i}{\dot{W}}, \tag{7a}$$

$$\sum_{i=1}^{2} \alpha_i = 1, \tag{7b}$$

where α_i is the ratio of the power $\dot{W}_i$, dissipated in the drilling ($i=1$) or wear ($i=2$) processes, to the total power supplied $\dot{W}$. As a consequence, Eq. (6) becomes

$$S_i = \alpha_i \frac{\mu H_i}{k_i}, \tag{8}$$

H_i and k_i being the hardness and the wear coefficient for the base ($i=1$) and tool ($i=2$) materials. As a first approximation, for a multiphase material, they can be obtained from the usual rules of mixture (Horbogen, 1986; Zum-Gahr, 1987)

$$H_i = \sum_j v_j^{(i)} H_j^{(i)}, \tag{9}$$

$$k_i = \sum_j v_j^{(i)} k_j^{(i)}, \tag{10}$$

$v_j^{(i)}$ being the volumetric percentage of the phase j in the composite material i, and $H_j^{(i)}$, $k_j^{(i)}$ the corresponding hardness and wear coefficient. Thus, by Eq. (10) different materials or cut configurations in reinforced concrete (e.g., “central” and “banana” cuts, see Figure 1) can be investigated.

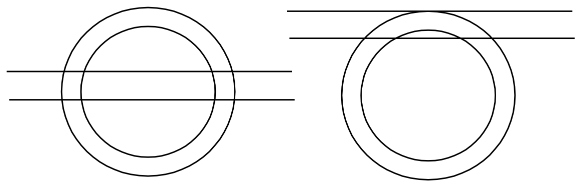

Figure 1. Central and banana-cut re-bar configurations.

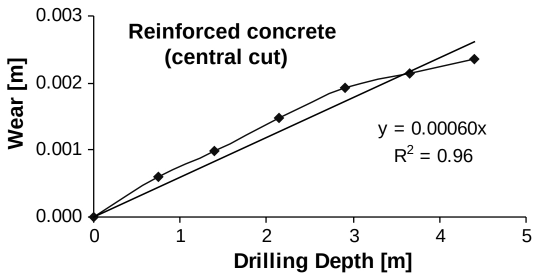

Figure 2. Wear vs. drilling depth for reinforced concrete in central cut configuration.

Table 1. Experimental results of wear and drilling depths.

Material	Reinforced concrete central cut	Reinforced concrete banana cut	Mortar	Concrete 1	Concrete 2
$\frac{\delta_2}{\delta_1} \cong \frac{\dot{\delta}_2}{\dot{\delta}_1}$	60×10^{-5}	43×10^{-5}	9×10^{-5}	3×10^{-5}	60×10^{-5}

Equation (8) is useful, permitting us to obtain theoretically the ratio between wear and drilling velocities (coupled parameter)

$$\frac{\alpha_2}{\alpha_1} \equiv \frac{\dot{W}_2}{\dot{W}_1} = \frac{S_2 A_2 \dot{\delta}_2}{S_1 A_1 \dot{\delta}_1} = \frac{\alpha_2 \mu H_2}{k_2} \frac{k_1}{\alpha_1 \mu H_1} \frac{A_2 \dot{\delta}_2}{A_1 \dot{\delta}_1} = \frac{\alpha_2 H_2 k_1 A_2 \dot{\delta}_2}{\alpha_1 H_1 k_2 A_1 \dot{\delta}_1}, \tag{11}$$

where A_1, A_2 are the area of the ring hole and of the segments, respectively, ($A_1 \cong A_2$) and $\dot{\delta}_1, \dot{\delta}_2$ are the drilling and wear velocities ($\dot{V}_i = A_i \dot{\delta}_i$), from which the Coupled Law becomes

$$\frac{\dot{\delta}_2}{\dot{\delta}_1} = \frac{H_1 k_2 A_1}{H_2 k_1 A_2} = \text{constant} \tag{12}$$

and predicts a linear relationship between wear and drilling velocities, as a function of classical parameters like hardness and wear coefficient of base and tool materials. This law can be considered only a first approximation of the physical reality (since it neglects the multi-scale and fractal character of the energy dissipation, see Section 3), as shown by our experimental results (Figures 2–6 and Table 1). The tests were performed on tool segment profiles by a computer aided optical system. The experimental apparatus is able to take into account the volumetric wear rate.

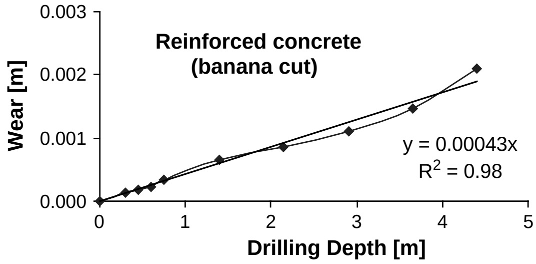

Figure 3. Wear vs. drilling depth for reinforced concrete in banana cut configuration.

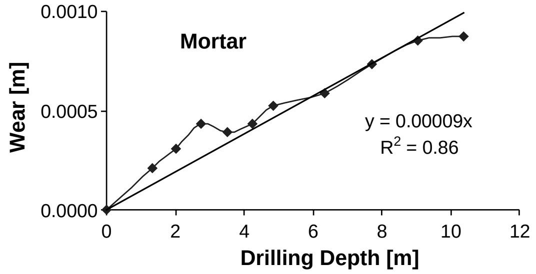

Figure 4. Wear vs. drilling depth for mortar.

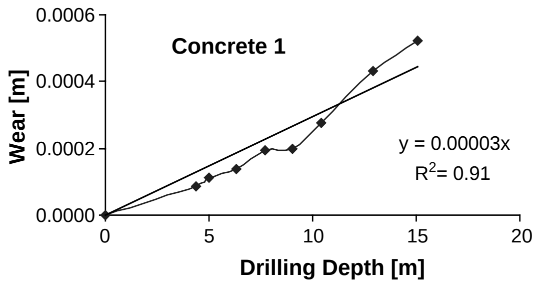

Figure 5. Wear vs. drilling depth for concrete 1.

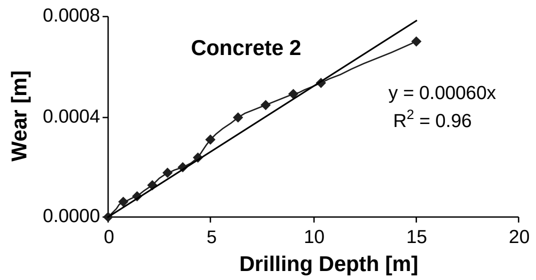

Figure 6. Wear vs. drilling depth for concrete 2.

Even if Eq. (12), as emphasized, represents a simplification of the reality, it allows one to solve the coupled problem. We have in fact

$$\dot{W}_i = \alpha_i \dot{W} = S_i A_i \dot{\delta}_i = \alpha_i \frac{\mu H_i}{k_i} A_i \dot{\delta}_i \tag{13}$$

and, eliminating α_i

$$\dot{\delta}_i = \frac{k_i \dot{W}}{\mu A_i H_i}. \tag{14}$$

It is important to emphasize that, in Eq. (14), $\dot{W}$ is the total (measurable) power and not the unknown power fractions dissipated in the wear or drilling processes. This equation can be used to evaluate both the wear and drilling velocities. Usually, the drilling and wear resistances (as well as the wear coefficient) can be considered constant parameters. Substantially, this is a consequence of the classical wear theory proposed by Reye (1860) 140 years ago and universally accepted, for which the removed volume is proportional to the energy dissipation (see also Villaggio, 2001). As a consequence, drilling and wear velocities become proportional to the power consumption. For a better description, we have to take into account the multi-scale and fractal character of the phenomenon. A "geometrical" multifractal extension has been also proposed by the same authors (Carpinteri and Pugno, 2003).

3. Fractal coupled law of wear and drilling velocities

In this section, a multi-scale and fractal theory, extending the classical concepts developed in the previous one, is presented. This theory is substantially based on the fractal universal law for energy dissipation during fragmentation (Carpinteri and Pugno, 2002b), that can be here summarized as

$$\dot{W} = \Gamma^* \dot{V}^\gamma, \quad 2/3 \leqslant \gamma \leqslant 1, \tag{15}$$

i.e., the power $\dot{W}$ dissipated in the comminution process is proportional to the fragmented volume per unit time $\dot{V}$ raised to a fractal exponent γ, comprised between 2/3 and 1. For wear it can be considered the generalisation of the Reye's hypothesis (Reye, 1860). As a matter of fact, the energy dissipation is classically assumed arising in a volume, so that $\gamma = 1$. On the other hand, if the dissipation arises on a surface, the fractal exponent becomes $\gamma = 2/3$. In general, it arises over a fractal domain comprised between a surface and a volume, so that $2/3 \leqslant \gamma \leqslant 1$. Γ^* is the so-called fractal fragmentation strength and appears to be a constant, differently from the usual fragmentation strength S. It is important to emphasize that its physical dimensions are $[F][L]^{1-3\gamma}[T]^{\gamma-1}$ and become those of a pressure only in the classical case of $\gamma = 1$. Therefore, we can define a fractal wear loss w^*, generalising the classical concept, as proportional to the energy dissipated in the comminution wear process, per unit area A, per unit length x of sliding and per unit time

$$w^* = \frac{\dot{V}^\gamma}{\dot{x} A}. \tag{16}$$

On the other hand, the fractal wear coefficient k^* can be defined as the probability of wear in the portion of the surface which is interacting

$$w^* = k^* \frac{A_{\text{contact}}}{A} = k^* \frac{F}{AH}. \tag{17}$$

where A_{contact} is the portion of the nominal area A in contact, H is the hardness (of the worn material) and F is the thrust. The main difference between the fractal wear loss w^* and the fractal wear coefficient k^* is that the first is not a material property being thrust-dependent. On the other hand, k^* can be considered as the real material constant with anomalous dynamic dimensions of $[L]^{3(\gamma-1)}[T]^{1-\gamma}$. It is important to emphasise that only in the classical case $\gamma = 1$ it is a dimensionless parameter.

The energy dissipated during a pure wear process can be obtained from Eq. (3). Eliminating F from Eqs. (3) and (17), we obtain

$$w^* = \frac{k^*}{AH\mu}\frac{\mathrm{d}W}{\mathrm{d}x} = \frac{k^*}{AH\mu}\frac{\dot{W}}{\dot{x}}. \tag{18}$$

From Eq. (16) and from the definition of fractal wear strength Γ^* (see Eq. (15)), we have

$$w^* = \frac{\dot{V}^{\gamma}}{\dot{x}A} = \frac{1}{\dot{x}A}\frac{\dot{W}}{\Gamma^*}. \tag{19}$$

Combining Eqs. (18) and (19), we obtain the fractal wear strength Γ^* as

$$\Gamma^* = \frac{\mu H}{k^*}. \tag{20}$$

The developed theory has facilitated the relationship between the fractal wear coefficient k^* and the fractal wear strength Γ^* (of the tool with hardness H). For γ tending to 1, Eq. (20) becomes Eq. (8) (energy dissipation assumed to occur in a volume).

If Eq. (20) were applied to the drilling process, Γ^* would represent the fractal drilling strength (of the base material). It is a macroscopic parameter (Carpinteri and Pugno, 2002, 2003) and has been previously obtained as a function of microscopic material constants, like the hardness H and the fractal wear coefficient k^* of the base material.

It is important to emphasize that relationship (20) is rigorous only for a pure wear (or drilling) process, i.e., we have assumed that the whole power is entirely dissipated, even if on a multi-scale domain – in wear (or drilling). This can not be assumed (by definition) in a coupled (e.g., drilling-wear) theory, for which the Equation (20) must be modified taking into account that not the whole power is dissipated in wear (or drilling). According to these considerations, Eq. (3) must be replaced with Eq. (7) and, consequently, Eq. (20) becomes

$$\Gamma_i^* = \alpha_i \frac{\mu H_i}{k_i^*}. \tag{21}$$

As a first approximation, for a multiphase material H_i can be obtained from the classical rule of mixture of Eq. (9) (Hornbogen, 1986; Zum-Gahr, 1987) and k_i^* from the fractal rule of mixture (Carpinteri and Pugno, 2004b):

$$k_i^* = \sum_j \left(v_j^{(i)}\right)^{\gamma_i} k_j^{*(i)}, \tag{22}$$

where $v_j^{(i)}$ being the volumetric percentage of the phase j in the composite material i (having fractal exponent γ_i), and $H_j^{(i)}$, $k_j^{*(i)}$ the corresponding hardness and fractal

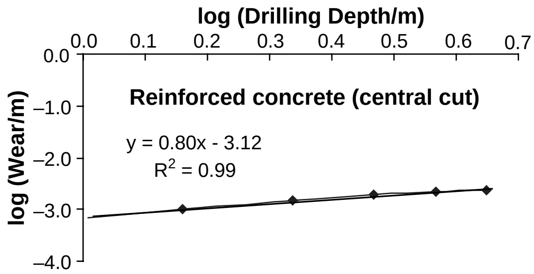

Figure 7. Bilogarithmic diagram wear vs. drilling depth for reinforced concrete in central cut configuration.

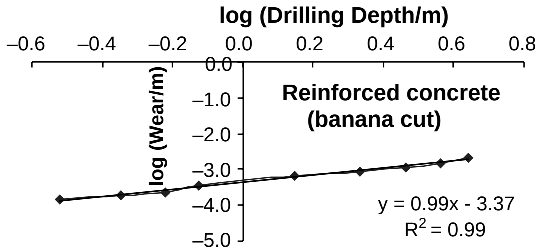

Figure 8. Bilogarithmic diagram wear vs. drilling depth for reinforced concrete in banana-cut configuration.

wear coefficient. Equation (21) is useful, permitting us to obtain theoretically the relationship between wear and drilling velocities (coupled parameter)

$$\frac{\alpha_2}{\alpha_1} \equiv \frac{\dot{W}_2}{\dot{W}_1} = \frac{\Gamma_2^* \left(A_2\dot{\delta}_2\right)^{\gamma_2}}{\Gamma_1^* \left(A_1\dot{\delta}_1\right)^{\gamma_1}} = \frac{\alpha_2 \mu H_2}{k_2^*} \frac{k_1^*}{\alpha_1 \mu H_1} \frac{\left(A_2\dot{\delta}_2\right)^{\gamma_2}}{\left(A_1\dot{\delta}_1\right)^{\gamma_1}} = \frac{\alpha_2 H_2 k_1^* \left(A_2\dot{\delta}_2\right)^{\gamma_2}}{\alpha_1 H_1 k_2^* \left(A_1\dot{\delta}_1\right)^{\gamma_1}} \tag{23}$$

from which the Fractal Coupled Law becomes

$$\frac{\dot{\delta}_2^{\gamma_2}}{\dot{\delta}_1^{\gamma_1}} = \frac{H_1 k_2^* A_1^{\gamma_1}}{H_2 k_1^* A_2^{\gamma_2}} \tag{24}$$

and takes into account the coupling of the multi-scale energy dissipations over fractal domains. It is a power-law relationship; since $2/3 \leqslant \gamma_i \leqslant 1$, its limit cases are

$$\dot{\delta}_2 \propto \dot{\delta}_1^{2/3}, \tag{25a}$$

$$\dot{\delta}_2 \propto \dot{\delta}_1^{3/2}, \tag{25b}$$

i.e., the exponent of the drilling velocity is theoretically comprised between 2/3 and 3/2.

The Fractal Coupled Law agrees well with the experimental results on impregnated diamond drilling tests performed by ourself and other authors (Miller and Ball, 1990, 1991; Tian and Tian, 1994), showing that the main parameter influencing the wear of the tool is the drilling velocity.

We have performed some experimental tests on tool segment profile by a computer aided optical system. The corresponding wear as a function of the drilling depth confirms fractal exponents comprised between the limit cases of mortar (0.68) and concrete (1.43), for all the materials tested and tools utilized (Figures 7–11), according

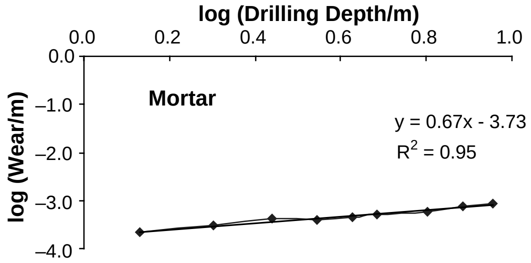

Figure 9. Bilogarithmic diagram wear vs. drilling depth for mortar.

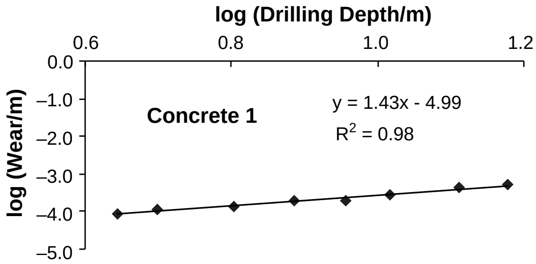

Figure 10. Bilogarithmic diagram wear vs. drilling depth for concrete 1.

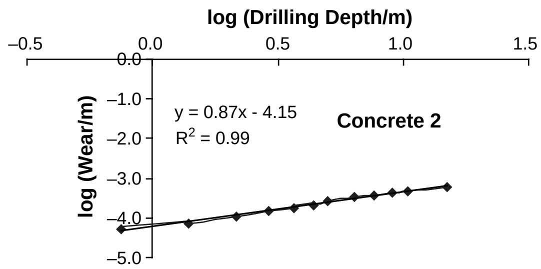

Figure 11. Bilogarithmic diagram wear vs. drilling depth for concrete 2.

Table 2. Experimental results of wear and drilling fractal exponents.

Material	Reinforced concrete central cut	Reinforced concrete banana cut	Mortar	Concrete 1	Concrete 2
$\frac{\gamma_1}{\gamma_2}$	0.80	0.99	0.67	1.43	0.87
$\frac{\delta_2}{(\delta_1)^{\frac{\gamma_1}{\gamma_2}}}$ $[\mathrm{m}]^{1-\frac{\gamma_1}{\gamma_2}}$	76×10^{-5}	43×10^{-5}	19×10^{-5}	1×10^{-5}	7×10^{-5}

to Eqs. (25). The experimental results are summarized in Table 2, where the slopes of the straight lines, γ_1/γ_2, and the values of their intercepts, $\delta_2/(\delta_1)^{\gamma_1/\gamma_2}$ are reported.

This allows one to solve the coupled problem. We have in fact

$$\dot{W}_i = \alpha_i \dot{W} = \Gamma_i^* \left(A_i \dot{\delta}_i\right)^{\gamma_i} = \alpha_i \frac{\mu H_i}{k_i^*} \left(A_i \dot{\delta}_i\right)^{\gamma_i} \tag{26}$$

and, from Eq. (24), the total power consumption becomes

$$\dot{W}=\Gamma_1^*\left(A_1\dot{\delta}_1\right)^{\gamma_1}+\Gamma_2^*\left(A_2\dot{\delta}_2\right)^{\gamma_2}=\left(\Gamma_1^*+\Gamma_2^*\left(\frac{H_1k_2^*}{H_2k_1^*}\right)\right)\left(A_1\dot{\delta}_1\right)^{\gamma_1}=\Gamma^*\left(A_1\dot{\delta}_1\right)^{\gamma_1}. \quad (27)$$

It is important to emphasize that, in Eq. (27), $\dot{W}$ is the total (measurable) power and not the unknown power fractions dissipated in the wear or drilling processes. Due to this reason, the process constant Γ^* can be determined just with one experiment; the constant Γ^* represents the fractal strength, size and drilling velocity independent, for the studied coupled drilling and wear phenomenon.

From Eq. (27), we can obtain a multi-scale prediction of the drilling velocity and Eq. (24) may provide the corresponding wear velocity.

4. Conclusions

The developed theory has permitted to predict the relationship between drilling and wear velocities, as described by the Fractal Coupled Law of equation (24), taking into account the coupling of the multi-scale energy dissipations over fractal volumes. The experiments on wear and drilling velocities, summarized in Figures 7–11 and in Table 2, agree well with the theoretical fractal predictions. By using Eqs. (24) and (27), we can obtain separately the drilling and wear velocities during perforations as functions of the total (measurable) power consumption. It has been emphasized how Eq. (15) can be considered as a generalisation of the classical Reye's assumption on wear (Reye, 1860).

The fractal coupled theory of drilling and wear describes the competition between the energy consumption in these two different processes. Fractal exponents γ around 1 would describe dissipations in a volume, as well as fractal exponents γ around 2/3 describe dissipations on a surface. Therefore, the ratio γ_1/γ_2 between the two fractal exponents depends on the domain (volume or surface or fractal set) in which the two energy dissipations arise.

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