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ORIGINAL PAPER

The unacknowledged risk of Himalayan avalanches triggering

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Abstract A "universal" model for avalanche triggering, as well as for collapse of suspended seracs, is presented based on Quantized Fracture Mechanics, considering fracture, friction, adhesion and cohesion. It unifies and extends the classical previous approaches reported in the literature, including the role of the slope curvature. A new size-effect, that on mountain height rather than the classical one on snow slab thickness, is also discussed and demonstrated thanks to glaciers data analysis from the World Glacier Inventory (http://nsidc.org/data/glacier_inventory/browse.html, 2014). The related most noteworthy result is that snow precipitation needed to trigger avalanches at 8,000 m could be up to 4 times, with a realistic value of 1.7 times, smaller than at 4,000 m. This super-strong size-effect may suggest that the risk of Himalayan avalanches is today still unacknowledged. A discussion on the recent Manaslu tragedy concludes the paper.

Keywords Avalanche · Triggering · Size-effects

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1 Introduction

On September 23, 2012 a massive avalanche hit Camp (C) 3, at 7,000 m, on the Manaslu mountain, see Fig. 1. It completely destroyed C3 and even wiped out tents in C2, much further down the slope. At the time, there were approximately 25 people in C3, 11 of which died. C3 was previously considered to be in an acceptably safe position. Professional mountaineers are well aware of risks associated with climbing, but one risk has never been discussed and is probably still unacknowledged (Pugno et al. 2013). We are referring to the difference between avalanche triggering at 4,000 and 8,000 m. Size-effects, not those classical related to slab thickness (McClung 1979) but rather connected to mountain height and never discussed in the literature, might have been responsible for the underestimated risk in the position of C3, perhaps due to the much greater experience cumulated by alpinists on 4,000 m mountains rather than at 8,000 m. This tragedy has prompted the following analysis, already discussed with Himalayan Alpinists in the Alpine Journal for its 150-year anniversary (Pugno et al. 2013). The related most noteworthy result of the new "universal" model presented here, is that snow precipitation needed to trigger avalanches at

Fig. 1 Manaslu tragedy: the zone of avalanche triggering, (photograph courtesy of Christian Gobbi)

8,000 m could be up to 4 times smaller than at 4,000 m. Our model is an extension of the classical approaches proposed by McClung (1979), previously introduced for over-consolidated clay by Palmer and Rice (1973), used by Bazant and McClung (2003) for predicting the size-effects on slab thickness, and subsequently extended -for considering non rigid interfaces- by Chiaia et al. (2008). The role of the slope curvature and tensile fracture, previously ignored in the literature, of the snow slab are also demonstrated to be not negligible.

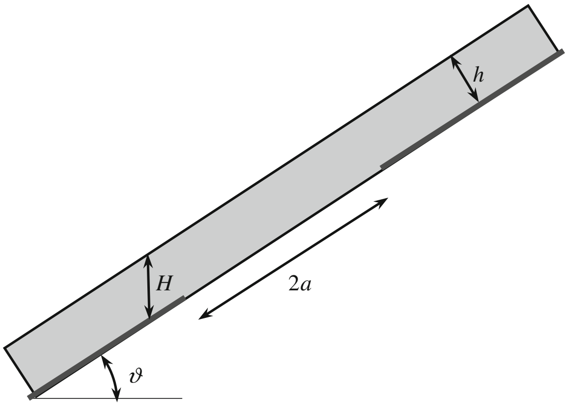

Fig. 2 Scheme of the Quantized Fracture Mechanics model of avalanche triggering or serac collapse

2 The "universal" model for avalanche triggering and serac collapse

The avalanche triggering and serac collapse can be predicted according to Quantized Fracture Mechanics (QFM) (Pugno 2006), generalizing the classical approach including friction, adhesion and, for removing the paradox of an infinite shear strength at vanishing crack length, a quantized crack advancement. If a defect of length $2a$ is present in the weak layer, the so called super-weak zone, see Fig. 2, an axial force $N(x)$ will occur in the debonded portion of the snow slab/serac; for arguments of symmetry the normal force must vanish at the centre of the crack (McClung 1979; Palmer and Rice 1973; Bazant and McClung 2003; Chiaia et al. 2008; Pugno and Carpinteri 2003), and can be calculated according to the following force equilibrium, see Fig. 2, of an infinitesimal element of length dx (Chiaia et al. 2008; Pugno and Carpinteri 2003; Kostantinidis et al. 2009; Pugno et al. 2011):

$$\tau(x) + \left(\tau_f + \tau_a\right) + \frac{\mathrm{d}N}{w\mathrm{d}x} - \tau_N = 0 \tag{1}$$

where

$$\tau_N = \rho g H \sin\vartheta \cos\vartheta \tag{2}$$

is the applied shear stress imposed by the weight of the snowfall of (vertical) height H or of the serac of (perpendicular) thickness (see Fig. 2):

$$h = H \cos\vartheta \tag{3}$$

and snow/ice density ρ, on a slope ϑ, where g is the acceleration of gravity, and:

$$\tau_f = f \rho g H \cos^2\vartheta \tag{4}$$

is the resistant shear stress of friction, with f friction coefficient of the sliding interface, τ_a is the resistant shear stress of adhesion [thus $\tau_f + \tau_a$ represents the residual shear strength after fracture and this is the reason of its appearance in Eq. (1)], $\tau(x)$ is the shear stress exchanged between substrate and slab and w is the snow slab/serac width.

In the debonded region $\tau(x) = 0$ and thus we derive:

$$N(x) = w \int_0^x \left[\tau_N - \left(\tau_f + \tau_a\right)\right] \mathrm{d}x = \left[\tau_N - \left(\tau_f + \tau_a\right)\right] wx \tag{5}$$

Note that in the debonded zone the upper part of the snow slab/serac is in tension, whereas the lower one is compressed, Fig. 2.

In the bonded region, the compatibility implies:

$$N = E' h w \frac{\mathrm{d}u}{\mathrm{d}x} \tag{6a}$$

and

$$\tau = -G \frac{u}{t} \tag{6b}$$

where E' is the effective Young modulus for plane strain conditions, i.e.,

$$E' = E \Big/ \left(1 - v^2\right) \tag{7}$$

with E Young modulus and v Poisson coefficient of the snow/ice, G is the shear elastic modulus of the interface, of thickness t and u is the axial displacement. Inserting Eqs. (6) into (1) yields (Chiaia et al. 2008; Pugno and Carpinteri 2003; Kostantinidis et al. 2009; Pugno et al. 2011):

$$\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} - \frac{u}{c^2} = \frac{\tau_N - \left(\tau_f + \tau_a\right)}{E' h} \tag{8a}$$

where

$$c = \sqrt{\frac{E' h t}{G}} \tag{8b}$$

Solving Eqs. (8) with the imposed boundary conditions for the derivative of the displacement (i.e. for the axial force, known at the crack tips and negligible at the ends of the slab, here assumed to be sufficiently long with respect to its depth h) yields (Chiaia et al. 2008; Pugno and Carpinteri 2003; Kostantinidis et al. 2009; Pugno et al. 2011):

$$u(x) = -\frac{c^2}{E' h} \left[\tau_N - \left(\tau_f + \tau_a\right)\right] \left[1 + \frac{a}{c} e^{-\frac{|x-a|}{c}}\right], \quad |x| > a \tag{9a}$$

and thus

$$\tau(x) = \left[\tau_N - \left(\tau_f + \tau_a\right)\right] \left[1 + \frac{a}{c} e^{-\frac{|x-a|}{c}}\right], \quad |x| > a \tag{9b}$$

$$N(x) = aw \left[\tau_N - \left(\tau_f + \tau_a\right)\right] e^{-\frac{|x-a|}{c}}, \quad |x| > a \tag{9c}$$

Note that the axial force is zero in the bonded region for vanishing crack length, whereas tends to the constant $N = aw\left[\tau_N - \left(\tau_f + \tau_a\right)\right]$ for vanishing thickness t, c thus represents the length of the zone influenced by the presence of the crack.

The strain energy stored in the debonded part of snow slab/serac is (Chiaia et al. 2008; Pugno and Carpinteri 2003; Kostantinidis et al. 2009; Pugno et al. 2011):

$$\Phi_d(x) = 2 \int_0^a \frac{N^2}{2E' h w} \mathrm{d}x = \frac{w a^3}{3E' h} \left[\tau_N - \left(\tau_f + \tau_a\right)\right]^2 \tag{10}$$

whereas that in the bonded region is:

$$\Phi_b(x) = 2 \int_a^\infty \frac{N^2}{2E' h w} \mathrm{d}x = \frac{w a^2 c}{2E' h} \left[\tau_N - \left(\tau_f + \tau_a\right)\right]^2 \tag{11}$$

and finally that stored in the interface is:

$$\Phi_i(x) = 2 \int_0^\infty \frac{\tau(x)^2}{2G} t w \mathrm{d}x = \frac{tw}{G} \left[\tau_N - \left(\tau_f + \tau_a\right)\right]^2 \left(a + \frac{a^2}{2c}\right) \tag{12}$$

According to QFM (Pugno 2006) the avalanche triggering or serac collapse will take place when:

$$G^* = \frac{\Delta\Phi}{\Delta S} = G_C \tag{13}$$

where $\Delta S = 2w\Delta a$ is the quantized increment of crack surface area, Δa is the "fracture quantum", G_C is the fracture energy per unit area of the sliding interface (for mode II crack propagation) and:

$$\Phi = \Phi_d + \Phi_b + \Phi_i = \frac{w}{E'h}\left[\tau_N - \left(\tau_f + \tau_a\right)\right]^2 \left(\frac{a^3}{3} + a^2 c + ac^2\right) \tag{14}$$

is the total strain energy. Accordingly, we find the critical condition (13) for:

$$\tau_N = \tau_f + \tau_a + \tau_F \tag{15}$$

with:

$$\tau_F = \sqrt{\frac{E/\left(1-v^2\right)G_C H\cos\vartheta}{\left(3a^2 + 3a\Delta a + \Delta a^2\right)/6 + c(2a+\Delta a)/2 + c^2/2}} \tag{16}$$

that is the resistant shear stress of fracture. Note that Eq. (15) with Eqs. (16), (2) and (4) recovers the classical models by McClung 1979; Palmer and Rice 1973; Bazant and McClung 2003 for $\Delta a = \tau_f = \tau_a = c = 0$, by Kostantinidis et al. (2009), Pugno et al. (2011) for $\tau_f = \tau_a = c = 0$ and by Chiaia et al. (2008) for $\tau_f = \tau_a = \Delta a = 0$. Thus our "universal" model represents a generalization of the classical approaches reported in McClung (1979), Palmer and Rice (1973), Bazant and McClung (2003), Chiaia et al. (2008), Kostantinidis et al. (2009), Pugno et al. (2011). Assuming $\Delta a \neq \Delta a\,(H)$ (e.g. $\Delta a = 0$, as assumed in classical and thus continuum fracture mechanics or $\Delta a = const$ as considered in Kostantinidis et al. (2009), Pugno et al. (2011)) would allow us to solve the quadratic equation of the model in closed form and thus derive the critical snowfall height $H = H_C\,(\vartheta)$ or serac size $h = h_C\,(\vartheta)$ as a function of the slope.

In the present model we further consider:

$$\Delta a = \frac{-3c + \sqrt{9c^2 - 24\left\{c^2 - EG_C\cos\vartheta\, H/\left[\left(1-v^2\right)\tau_C^2\right]\right\}}}{2} \tag{17a}$$

physically derived imposing:

$$\tau_C = \tau_F\,(a=0) \tag{17b}$$

that is the resistant shear stress of cohesion, i.e. the ideal material strength in absence of defects. Also in this more complex case, equation (15) can be easily solved, e.g. iteratively even analytically or numerically.

3 Size-effects on mountain height

According to the classical hypothesis of self-similarity, commonly used in fracture mechanics and that assumes the larger the structure the larger the largest crack, we expect:

$$a \approx \varepsilon_1 l^{k_1} \tag{18a}$$

where l is the size of the slope and strictly speaking (if pure self-similarity is valid) $k_1 = 1$. The corresponding strength scaling law is predicted according to Griffith to be $\sigma \propto a^{-1/2} \propto l^{-k_1/2}$; according to a fractal scaling Carpinteri and Pugno (2005) one would instead predict $\sigma \propto l^{-(3-D)/2}$, where $2 \leq D \leq 3$ is the fractal dimension of the domain in which the energy is dissipated during fracture; comparing the two scaling laws we derive $0 \leq k_1 = 3 - D \leq 1$; since for most materials $D \approx 2.5$ (Carpinteri and Pugno 2005) we roughly estimate $k_1 \approx 1/2$, that is the intermediate case.

Since geometrically Himalayan avalanches can be much larger than those on the Alps, e.g. that on Manaslu, we statistically expect that the higher the mountain the larger the slope and thus:

$$l \approx \varepsilon_2 A^{k_2} \tag{18b}$$

where A is the mountain size/height and $0 \leq k_2 \leq 1$, whereas for pure self-similarity $k_2 = 1$. Note that the highest mountain face in the Alps is the Est Face (vertical length of 2,600 m) of Monte Rosa (altitude of 4,638 m) whereas in Himalayas is the Rupal Face (vertical length of 4,600 m) of Nanga Parbat (altitude of 8,126 m) and the ratios between the corresponding altitude of the mountain and length of the face are very similar and, respectively equal to 1.78 and 1.77, thus suggesting $k_2 \approx 1$. In order to better derive an estimation for k_2 we consider the lengths and altitudes of mountain glaciers from the World Glacier Inventory:

on Alps 1097 glaciers are reported for which we calculate a mean altitude of 2,936 m and a mean length of 1.3 km, whereas for Himalaya (India River and Gange River Zones) 10,799 glaciers are reported with a mean altitude of 5,588 m and a mean length of 2.0 km; accordingly we fit $k_2 = 0.75$ and thus we estimate $k_2 \approx 3/4$.

Combining Eqs. (18a,b) we thus find:

$$a \approx \varepsilon_1 \varepsilon_2 A^{k_1 k_2} = \varepsilon A^k \tag{18c}$$

where $0 \leq k \leq 1$ and a realistic estimation, according to the previous ones, is $k \approx 3/8$. Obviously this numerical estimation has to be considered with caution and the limiting case of $k = 1$ is more cautelative even if less realistic. The important point here is that k is different from 0 and thus the common perception of $k = 0$ and thus absence of size-effects is not physical. Accordingly, the critical condition or scaling law is predicted inserting Eqs. (17a) and (18c) into Eq. (16) and then Eqs. (2), (4) and (16) into Eq. (15). For example, in the limiting case of $c = 0$, we find:

$$\rho g h \sin\vartheta = f \rho g h \cos\vartheta + \tau_a + \sqrt{\frac{bh}{3\varepsilon^2 A^{2k} + 3\varepsilon A^k \frac{\sqrt{bh}}{\tau_C} + \frac{bh}{\tau_C^2}}} \tag{19}$$

with $b = 6EG_C/(1-v^2)$.

Equation (19) asymptotically predicts $h \propto H \propto A^{-2\tilde{k}}$ with a vanishing slope $\tilde{k} = 0$ for both limiting conditions of $A \to 0, \infty$ (in contrast to classical scaling laws that alternatively predict a vanishing slope only for $A \to 0$ or $A \to \infty$ but in perfect agreement with the general scaling law proposed in Pugno (2007) and an asymptotic matching with a maximal slope $\tilde{k} = k$. This limiting size-effects are not affected by the presence of a non zero value of c (that only implies a different scaling for $A \to 0$):

$$h \propto H \propto A^{-2k} \tag{20}$$

Accordingly, comparing 8,000 versus 4,000 peaks, we expect a limiting size-effect of:

$$H_C^{(8000)} = H_C^{(4000)} / 4^k \tag{21a}$$

$$h_C^{(8000)} = h_C^{(4000)} / 4^k \tag{21b}$$

The common perception would naively assume $k = 0$, thus no size-effect, whereas for $k = 3/8$:

$$H_C^{(8000)} = H_C^{(4000)} / 1.7 \tag{22a}$$

$$h_C^{(8000)} = h_C^{(4000)} / 1.7 \tag{22b}$$

and for the limiting case of pure self-similarity:

$$H_C^{(8000)} = H_C^{(4000)} / 4 \tag{22c}$$

$$h_C^{(8000)} = h_C^{(4000)} / 4 \tag{22d}$$

i.e. a super-strong size-effect, by force in contrast to the common perception ($k = 0$). Of course this represents a limiting condition. This has a remarkable implication, see Fig. 3a, b ($\tau_N = \tau_F\ (\Delta a, c = 0, k = 1)$). Below the lower curve the conditions are safe even on a 8,000 peak whereas above the upper curve the conditions are unsafe even on a 4,000 peak. The most dangerous zone is that between the two curves: presence of "abnormal" conditions that dangerous at 8,000 m, but that experi-

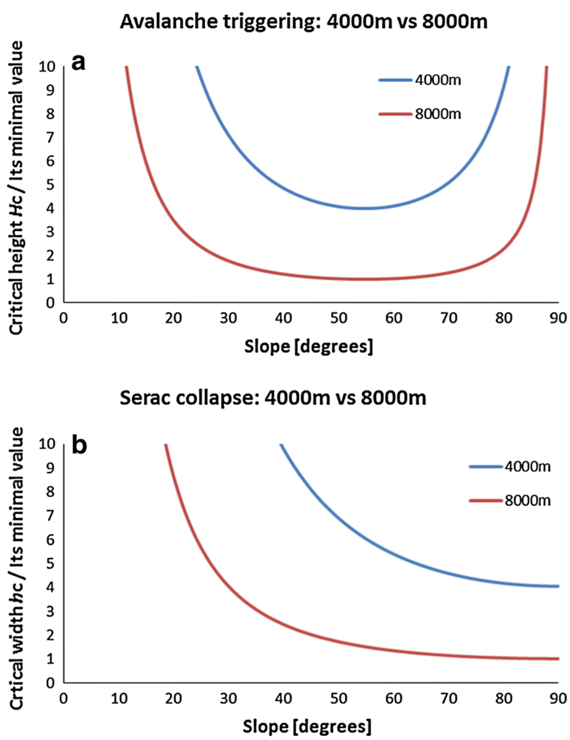

Fig. 3 Limiting size-effects ($\tau_N = \tau_F\ (\Delta a, c = 0, k = 1)$) on **a** the dimensionless critical height of snowfall necessary to cause the detachment of an avalanche (H_C) or **b** on the dimensionless width of the critical part of suspended serac necessary to cause it to collapse (h_C): 4,000 versus 8,000 m predictions. Below the *lower curve* the conditions are safe even on a 8,000 peak. Above the *upper curve* the conditions are unsafe even on a 4,000 peak. The most dangerous zone is that between the two curves: presence of "abnormal" conditions that dangerous at 8,000 m, but that experience would rightly lead to considering safe at 4,000 m experience on Alps

ence would rightly lead to considering safe at 4,000 m experience on Alps.

4 The role of the curvature of the slope and of the stress redistribution after tensile (or compressive) collapse of the snow/ice

The curvature of the mountain slope, previously ignored in the literature, is demonstrated here to play an important role. Instead of rewriting the theory reported in the previous section for curved slope we derive in this section a simple first order correction.

The indefinite force equilibrium equations for a curved beam are:

$$\frac{\mathrm{d}N}{\mathrm{d}s}+\frac{T}{r}+p=0,\quad \frac{\mathrm{d}T}{\mathrm{d}s}-\frac{N}{r}+q=0 \tag{23}$$

where N and T are the normal and shear forces as well as p and q are the applied axial or normal loads per unit length; r is the radius of curvature of the slope. Accordingly the following equation holds:

$$\frac{\mathrm{d}^2N}{\mathrm{d}s^2}+\frac{1}{r}\left(\frac{N}{r}-q\right)+T\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{r}\right)+\frac{\mathrm{d}p}{\mathrm{d}s}=0 \tag{24}$$

Assuming small ($\frac{N}{r^2}\approx 0$) and nearly constant $\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{r}\right)\approx 0$ curvature, we find:

$$\frac{\mathrm{d}^2N}{\mathrm{d}s^2}-\frac{q}{r}+\frac{\mathrm{d}p}{\mathrm{d}s}=0 \tag{25}$$

Thus the following relationship emerges:

$$\frac{\mathrm{d}p\,(r=\infty)}{\mathrm{d}s}=-\frac{q}{r}+\frac{\mathrm{d}p\,(r)}{\mathrm{d}s} \tag{26}$$

Since $\frac{p}{q}=\tan\vartheta$ we have:

$$p\,(r)\approx p\,(r=\infty)\left(1+\frac{l}{r\tan\vartheta}\right) \tag{27}$$

where l is the length of the slope, and thus finally:

$$\tau_N\,(r)\approx\tau_N\,(r=\infty)\left(1+\frac{l}{r\tan\vartheta}\right) \tag{28}$$

Equation (28) clearly shows that the radius of curvature of the slope can increase ($r>0$, convex, snow more tensioned) or decrease ($r<0$, concave, snow more compressed) the effective applied load and this effect is weakened for slope tending to the vertical condition (as it must be for symmetry).

Finally note that if the maximum tensile or compressive stress in the snow:

$$\sigma\,(x=\mp a)=\mp\left[\tau_N-\left(\tau_f+\tau_a\right)\right]\frac{a}{h} \tag{29}$$

reaches the snow material strength in traction or compression, σ_T, σ_C, respectively, then the transversal collapse of the snow/ice takes place; after tensile fracture (at $x=a$) the axial load is linearly redistributed from 0 up to a maximal value (at $x=-a$) that is doubled than that reported in Eq. (29), thus the tensile fracture further weakens the stability of the snow. The new triggering condition can be calculated according to the previous analysis where fictitiously the crack length becomes doubled. Note that the upper part of the snow slab is not any more loaded by an axial force whereas the axial force in the lower part is doubled. This evolutive mechanism was not previously discussed in the literature.

5 Discussion on the Manaslu tragedy

As reported in the Introduction, the Manaslu avalanche hit camp number 3, at about 7,000 m altitude. The mountaineer Silvio "Gnaro" Mondinelli, who survived the tragedy, personally told us that the avalanche may have been triggered by a falling serac and that the slope, covered in about 3 m of snow, was around 50° (Pugno et al. 2013). It is true that an avalanche or a collapse of a serac can take place at both altitudes of 4,000 and 8,000 m, but at 8,000 we have demonstrated that there is an additional risk of which even the very thin air mountaineers have never heard of (Pugno et al. 2013). Let us thus refer to the general model here derived and consider some relevant limiting cases.

The simplest model to derive the propagation of an avalanche predicts the detachment by friction when the shear stress on the interface with the weakest layer (typically consisting of snow crystals of larger size) reaches a certain critical value, given by the pressure of the snow multiplied by the friction coefficient. In this model ($\tau_N=\tau_f$), detachment is predicted independent of the amount of accumulated snow and at a slope inclination angle equal or greater than the angle of friction (arctan of the friction coefficient). A different model predicts detachment when the shear stress, imposed by the accumulation of snow, reaches a critical constant value which is characteristic of the material strength ($\tau_N = \tau_a = \tau_C$). In this model, the detachment of the avalanche is possible for any slope, as long as there has been a sufficiently abundant precipitation. The least favourable slope, corresponding to the minimum necessary precipitation to cause detachment, occurs at 45° (whereas, mathematically, at 0° and 90° the necessary precipitation to cause detachment tends to infinity). A more evolved third model is based on classical fracture mechanics ($\tau_N = \tau_f$ ($\Delta a = 0$)). Our "universal" model, see Eqs. (15) and (16), is an extension of the existing classical approaches and can also be applied for the calculation of the collapse of suspended seracs (and rock avalanches, i.e. landslides). It takes into account friction, adhesion, cohesion and fracture and also has the great advantage (also present in classical fracture mechanics) to be sufficiently realistic as to highlight the size scale of the most dangerous defect that generates detachment or collapse. It is not easy to identify in practice this defect (it may be the whole weak interface zone or a super-weak portion of it), let alone its size, but it is reasonable to assume that it is proportional to the size of the slope or the serac, which in turn are proportional to the height of the mountain, see Eq. (18c). Accordingly, we have demonstrated the presence of "abnormal" conditions that dangerous at 8,000 m, but that experience would rightly lead to considering safe at 4,000 m, i.e. configurations between the two curves in the graphs of Fig. 3, see Eq. (21). Scientists, Engineers and Himalayan mountain climbers must thus keep into account such size-effects and related risks, today unacknowledged, when translating their experience at 4,000 m to conditions at 8,000 m. These have been the cause of the collapse of ships, bridges and entire buildings. There is no reason to believe that they are not at play in the mountains. They may have played an important role on Manaslu too.

6 Conclusions

A "universal" model for avalanche triggering, as well as for collapse of suspended seracs, has been presented, unifying and extending the classical previous approaches reported in the literature, including the role of the slope curvature, evolutive (tensile fracture + sliding) failure and size-effect on mountain height. This new size-effect, differently from the classical one on snow slab thickness, shows that snow precipitation needed to trigger avalanches at 8,000 m could be up to 4, (with a more realistic value of 1.7) times smaller than at 4,000 m. This super-strong size-effect may suggest that the risk of Himalayan avalanches is today still unacknowledged by alpinists (Pugno et al. 2013).

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