



Optimal Hierarchical Architectures to Maximize the Strength-to-Weight Ratio of Bi-Materials

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In this paper the strength-to-weight ratio of bi-material composites with hierarchy is analyzed. The results show that the strength-to-weight ratio of the bi-material composites increases with the increase of the number of hierarchical levels and could be optimized by choosing the appropriate width ratio between the two materials. This paper suggests new strategies for the design of bi-material composites which are capable of maximizing the strength under a fixed weight per unit area. Mimicking nature an application for the optimal design of the textile structures is given.

Keywords: Bi-Material Composites, Strength-to-Weight Ratio, Optimization, Hierarchy, Textile.

1. INTRODUCTION

As a kind of special composites, functionally graded materials (FGMs) have attracted many attentions in the past years.^{1–6} They possess a number of advantages that make them attractive in different applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, reduced residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced concentrations and stress intensity factors.⁷ The fracture behavior of FGMs plays an important role in the practical design.^{8–9} Thus re-entrant corners in functionally graded materials have been also considered.^{4, 10–11} At the same time, as a typical example of functionally graded materials (FGMs), the bi-material composites have been studied by many researchers in the past years,^{6, 11–16} especially in the field of biological materials. At each level of structural hierarchy, biological materials often exhibit hard mineral platelets embedded in a soft protein matrix.¹⁷ Accordingly, a lot of work has been done in this context including prediction of the strength, toughness and stiffness as well as the formulation of new mimic design methods.

The structural weight too plays a vital role.⁷ One of the typical weight efficient structures is the cellular structure and many optimizing methods are given in the literatures.^{18–22} Obviously, the strength-to-weight ratio in

the cellular structure is attractive. But how can we optimize the strength-to-weight ratio in a general solid? What is the relationship between its hierarchical architecture and its strength-to-weight ratio?

Based on the work on re-entrant corners by Carpinteri and Pugno,⁴ this paper mainly focuses on the special crack case in a periodically distributed bi-material plate. The relative strength-to-weight ratios of the edge cracked bi-material composites with and without hierarchy are analyzed. Accordingly, the optimal width ratio between the two materials is obtained at each hierarchical level.

The paper is composed by three main sections. In Section 2 the edge cracked periodically distributed bi-material composite without hierarchy is analyzed and an application for the optimal design of textile structures is reported. Section 3 focuses on the relative strength-to-weight ratio of the edge cracked periodically distributed bi-material composites with hierarchy. Finally, remarks are given in Section 4.

2. STRENGTH-TO-WEIGHT RATIO OF THE PERIODICALLY DISTRIBUTED BI-MATERIAL COMPOSITES WITH AN EDGE CRACK

In this section the relative strength-to-weight ratio of the bi-material plate with an edge crack is analyzed to find in which case a maximum of the relative strength-to-weight ratio exists. One application for textile is also reported.

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2.1. The Relative Strength-to-Weight Ratio

Here we suppose that the bi-material plate has a unit thickness, n pairs of the bi-material strips, and thus the width is $b = nD + nd = n(x + 1)d$, where $x = D/d$.

The strength for a FGM composite plate in tension with a re-entrant corner is:⁴

$$\hat{\sigma}_c^* = \frac{\hat{K}_{IC}^*}{b^\alpha \hat{f}^*} \quad (1)$$

in which \hat{K}_{IC}^* is the critical value of the stress intensity factor for the re-entrant corner with angle γ , \hat{f}^* is the generalized shape function and the power α is related to γ by the eigen-equation $(1 - \alpha) \sin(2\pi - \gamma) = \sin[(1 - \alpha)(2 - \pi)]$. For the special crack case ($\alpha = 0$) with the tip in the matrix (material A, Fig. 1(a)) the strength of the bi-material plate is:

$$\sigma = \frac{\hat{K}_{IC}}{\sqrt{b} \hat{f}} = \frac{\hat{K}_{IC}}{\sqrt{b} \hat{g}(f/g)} = \frac{\hat{K}_{IC} g}{\sqrt{b} \hat{g} f} \quad (2)$$

in which⁴

$$\hat{K}_{IC} = \hat{K}_{IC}^{(A)} \quad (0 < a < D) \quad (3)$$

$$f = 2 \left(\frac{a}{b}\right)^{1/2} - 0.4 \left(\frac{a}{b}\right)^{3/2} + 18.7 \left(\frac{a}{b}\right)^{5/2} - 38.5 \left(\frac{a}{b}\right)^{7/2} + 53.9 \left(\frac{a}{b}\right)^{9/2} \quad \left(\frac{a}{b} < 0.6\right) \quad (4)$$

$$g = \frac{1}{1 - a/b} \quad (5)$$

$$\begin{aligned} \hat{g} &= \frac{\int_A E(y) dA}{\int_{A_{lig}} E(y) dA} = \frac{nDE_A + ndE_B}{nDE_A + ndE_B - aE_1} \\ &= \frac{nE_A x + nE_B}{nE_A x + nE_B - (a/d)E_A} \\ &= \frac{E_A x + E_B}{(1 - a/b)E_A x + E_B - (a/b)E_A} \quad (0 < a < D) \quad (6) \end{aligned}$$

At the same time, the density ρ for the bi-material plate is:

$$\begin{aligned} \rho &= \rho_A \nu_A + \rho_B \nu_B = \rho_A \frac{nD}{nD + nd} + \rho_B \frac{nd}{nD + nd} \\ &= \frac{\rho_A x + \rho_B}{x + 1} \quad (7) \end{aligned}$$

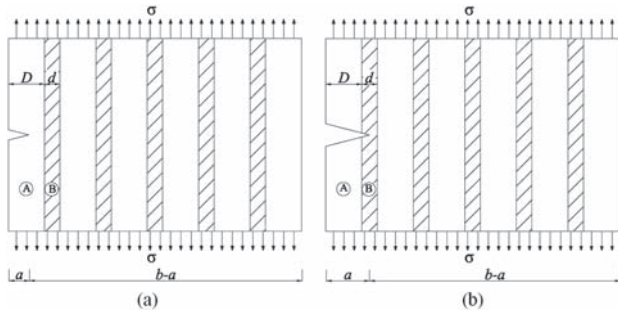


Fig. 1. Schematic of the bi-material plate with an edge crack.

in which ρ_A and ρ_B are the densities for material A and material B respectively.

Suppose the volume of the plate is V , then we get the strength-to-weight ratio for the bi-material plate with an edge crack in material A:

$$\begin{aligned} \frac{\sigma}{W} &= \frac{\sigma}{\rho \times V} = \frac{K_{IC}^{(A)} g}{\sqrt{b} f \rho_A V} \\ &\quad \times \frac{(1 - a/b)x^2 + (1 + z - (2a)/b)x + (z - a/b)}{x^2 + (y + z)x + yz} \\ &= \frac{K_{IC}^{(A)} g}{\sqrt{b} f \rho_A V} \frac{Ax^2 + Bx + C}{Hx^2 + Ex + F} \quad (0 < a < D) \quad (8) \end{aligned}$$

in which

$$\begin{aligned} y &= \rho_B / \rho_A, \quad z = E_B / E_A, \quad A = 1 - a/b, \\ B &= 1 + z - (2a)/b, \quad C = z - a/b, \quad H = (n + 1)^2, \\ E &= Y + z, \quad F = yz \end{aligned}$$

The strength-to-weight ratio for the same size homogeneous plate purely made of the matrix (material A) is

$$\frac{\sigma_A}{W_A} = \frac{K_{IC}^{(A)}}{\sqrt{b} f} / (\rho_A V) = \frac{K_{IC}^{(A)}}{\sqrt{b} f \rho_A V} \quad (9)$$

Comparing the above two results, i.e., Eqs. (8) and (9), we get the relative strength-to-weight ratio ρ_R / W_R of the bi-material plate for Figure 1(a):

$$\frac{\sigma_R}{W_R} = \frac{\sigma}{W} / \frac{\sigma_A}{W_A} = g \frac{Ax^2 + Bx + C}{Hx^2 + Ex + F} \quad (0 < a < D) \quad (10)$$

Similarly, when $d < a < D + d$, i.e., when the edge crack tip is in the strip composed by material B (Fig. 1(b)), $\hat{K}_{IC} = K_{IC}^{(B)}$, the relative strength-to-weight ratio ρ_R / W_R will be:

$$\begin{aligned} \frac{\sigma_R}{W_R} &= \frac{\sigma}{W} / \frac{\sigma_A}{W_A} \\ &= \frac{K_{IC}^{(B)}}{K_{IC}^{(A)}} g \frac{Ax^2 + Bx + C}{Hx^2 + Ex + F} \quad (d < a < D + d) \quad (11) \end{aligned}$$

2.2. Maximum Strength-to-Weight Ratio

The relative strength-to-weight ratio σ_R / W_R in Eq. (11) may display a maximum versus $x = D/d$. Accordingly, we search the solutions for

$$\begin{aligned} \frac{d}{dx} \left(\frac{\sigma_R}{W_R} \right) &= \frac{d}{dx} \left(\frac{\sigma/W}{\sigma_A/W_A} \right) \\ &= g \frac{d}{dx} \left(\frac{Ax^2 + Bx + C}{Hx^2 + Ex + F} \right) = 0 \quad (12a) \end{aligned}$$

that is

$$\frac{d}{dx} \left(\frac{Ax^2 + Bx + C}{Hx^2 + Ex + F} \right) = \frac{(AE - BH)x^2 + 2(AF - CH)x + BF - CE}{(Hx^2 + Ex + F)^2} = 0 \quad (12b)$$

It implies $(AE - BH)x^2 + 2(AF - CH)x + BF - CE = 0$ or $Hx^2 + Ex + F \rightarrow \infty$. Here we ignore the second case which will correspond to $\sigma/W \rightarrow 0$. From the first case it is easy to get the general solution for:

$$\begin{aligned} x &= \frac{D}{d} \\ &= \frac{-(AF - CH) \pm \sqrt{(AF - CH)^2 - (AE - BH)(BF - CE)}}{AE - BH} \\ &= \frac{-x_1 \pm \sqrt{x_1^2 - x_2x_3}}{x_2} \end{aligned} \quad (13a)$$

in which

$$\begin{aligned} x_1 &= AF - CH = \left(1 - \frac{a}{b}\right)yz - z + \frac{a}{b} \\ x_2 &= AE - BH = \left(1 - \frac{a}{b}\right)y - \frac{a}{b}z + 2\frac{a}{b} - 1 \\ x_3 &= BF - CE = (y - 1)z^2 2\frac{a}{b}yz + \frac{a}{b}y + \frac{a}{b}z \\ x_1^2 &= x_2x_3 = \frac{a}{b}(z - 1)^2(y - 1)\left(\frac{a}{b}y - y + z - \frac{a}{b}\right) \end{aligned} \quad (13b)$$

Considering the boundary conditions derived above for x we get the following conditions for the existence of the maximum:

$$\begin{cases} x_1^2 - x_2x_3 \geq 0 \\ x = \frac{-x_1 + \sqrt{x_1^2 - x_2x_3}}{x_2} > 0 \end{cases} \Rightarrow (1) \begin{cases} x_1 < 0 \\ x_2 > 0 \\ x_3 \geq 0 \\ x_1^2 \geq x_2x_3 \end{cases} \Rightarrow (1) \begin{cases} y < y_1 \\ y > y_2 \\ y \geq y_3 \\ 1 < y \leq y_4 \text{ or } y_4 \leq y < 1 \end{cases}$$

$$\Rightarrow (2) \begin{cases} x_2 > 0 \\ x_3 < 0 \end{cases} \Rightarrow (2) \begin{cases} y > y_2 \\ y < y_3 \end{cases} \quad (14a)$$

$$(3) \begin{cases} x_1 > 0 \\ x_2 < 0 \\ x_3 < 0 \\ x_1^2 \geq x_2x_3 \end{cases} \Rightarrow (3) \begin{cases} y > y_1 \\ y < y_2 \\ y < y_3 \\ 1 < y \leq y_4 \text{ or } y_4 \leq y < 1 \end{cases}$$

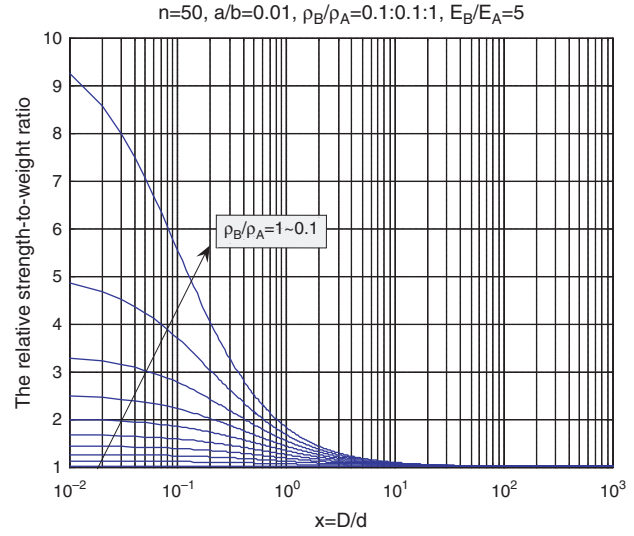


Fig. 2. The relative strength-to-weight ratio for $n = 50$, $a/b = 0.001$, $z = E_B/E_A = 5$ and $y = \rho_B/\rho_A = 0.1-1$.

$$\begin{cases} x_1^2 - x_2x_3 \geq 0 \\ x = \frac{-x_1 - \sqrt{x_1^2 - x_2x_3}}{x_2} > 0 \end{cases} \Rightarrow (4) \begin{cases} x_1 < 0 \\ x_2 > 0 \\ x_3 > 0 \\ x_1^2 \geq x_2x_3 \end{cases} \Rightarrow (4) \begin{cases} y < y_1 \\ y < y_2 \\ y < y_3 \\ 1 < y \leq y_4 \text{ or } y_4 \leq y < 1 \end{cases}$$

$$\Rightarrow (5) \begin{cases} x_2 < 0 \\ x_3 > 0 \end{cases} \Rightarrow (5) \begin{cases} y < y_2 \\ y > y_3 \end{cases}$$

$$(6) \begin{cases} x_1 > 0 \\ x_2 < 0 \\ x_3 \leq 0 \\ x_1^2 \geq x_2x_3 \end{cases} \Rightarrow (6) \begin{cases} y > y_1 \\ y < y_2 \\ y \leq y_3 \\ 1 \leq y \leq y_4 \text{ or } y_4 \leq y \leq 1 \end{cases} \quad (14b)$$

in which

$$\begin{aligned} y_1 &= \left(z - \frac{a}{b}\right) / \left[\left(1 - \frac{a}{b}\right)z\right] \\ y_2 &= \left(\frac{a}{b}z + 1 - 2\frac{a}{b}\right) / \left(1 - \frac{a}{b}\right) \\ y_3 &= \left(z^2 - \frac{a}{b}z\right) / \left(z^2 - 2\frac{a}{b}z + \frac{a}{b}\right) \text{ and} \\ y_4 &= \left(z - \frac{a}{b}\right) / \left(1 - \frac{a}{b}\right) \end{aligned}$$

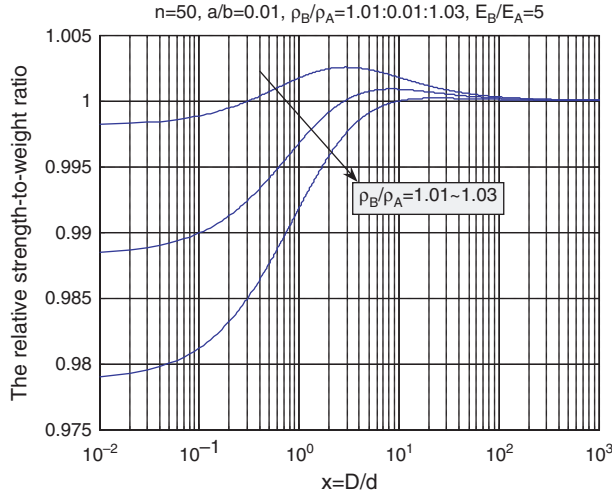


Fig. 3. The relative strength-to-weight ratio for $n = 50$, $a/b = 0.01$, $z = E_B/E_A = 5$ and $y = \rho_B/\rho_A = 1.01-1.03$.

If $z = E_B/E_A$ is fixed, we can choose $y = \rho_B/\rho_A$ for

$$x = \left(-x_1 + \sqrt{x_1^2 - x_2 x_3} \right) / x_2 > 0 \quad \text{or}$$

$$x = \left(-x_1 - \sqrt{x_1^2 - x_2 x_3} \right) / x_2 > 0$$

2.3. A Numerical Example

From $0 < a < D$ we have $0 < a/b < D/[nd(x+1)] = x/[n(x+1)] < 1/n$. Suppose $n = 50$, then $0 < a/b < 1/n = 0.02$. Here we suppose $a/b = 0.01$, $z = E_B/E_A = 5$ and $0.1 \leq y = \rho_B/\rho_A \leq 10$. From the previous section it is easy to verify that the maximum relative strength-to-weight ratio exists and the corresponding value for $y = \rho_B/\rho_A = 1.01, 1.02, 1.03$ is $x = D/d = 2.97, 8.50, 23.94$

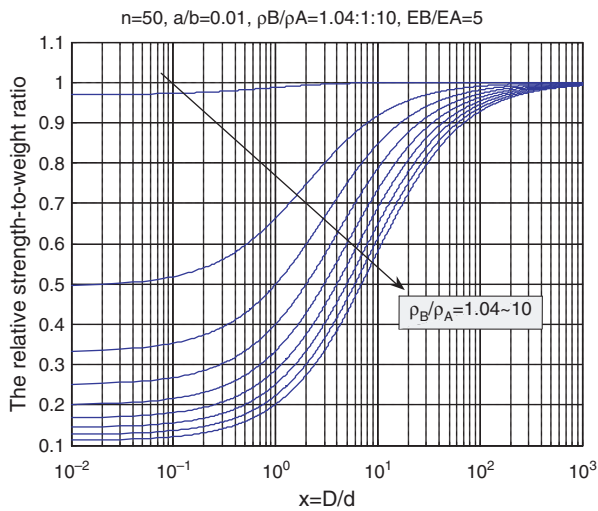


Fig. 4. The relative strength-to-weight ratio for $n = 50$, $a/b = 0.01$, $z = E_B/E_A = 5$ and $y = \rho_B/\rho_A = 1.04 \sim 10$.

respectively. In other cases of $y = \rho_B/\rho_A$ the relative strength-to-weight ratio does not display the maximum, see Figures 2–4.

From Figures 2–4 we can see that for $z = E_B/E_A = 5$, when $0.1 \leq y = \rho_B/\rho_A \leq 1$ the relative strength-to-weight ratio is decreasing versus $x = D/d$ while when $1.04 \leq y = \rho_B/\rho_A \leq 10$ it is increasing; only when $1.01 \leq y = \rho_B/\rho_A \leq 1.03$ the maximum strength-to-weight ratio exists. For other cases of $z = E_B/E_A$, the same method can be used for the analysis of the relative strength-to-weight ratio. A practical application of this theory is the optimal design of textile structures, see appendix.

3. ROLE OF HIERARCHY

In this section we focus on the role of hierarchy with the crack tip at material B, see Figure 5. The strength, density and the Young's modulus of the bi-material composites at hierarchical level N are respectively:

$$\sigma_n = \frac{\hat{K}_{IC} g}{\sqrt{b_N \hat{g} f}} = \frac{K_{IC}^B |_{N} g_N}{\sqrt{b_N f_N}} \times \frac{(n-1)E_A x + [(1 - (a_N/b_N)n)x + n(1 - a_N/b_N)]E_{N-1}}{nE_A x + nE_{N-1}} \quad (15)$$

$$\rho_N = \rho_A \frac{x}{x+1} + \rho_{N-1} \frac{1}{x+1} = \frac{\rho_A x + \rho_{N-1}}{x+1} \quad (16)$$

$$E_N = E_A \frac{x}{x+1} + E_{N-1} \frac{1}{x+1} = \frac{E_A x + E_{N-1}}{x+1} \quad (17)$$

The strength-to-weight ratio for the plate with the same volume, purely made by material A, is

$$\frac{\sigma_A}{W_A} \Big|_N = \frac{K_{IC}^{(A)}}{\sqrt{b_N f_N}} / (\rho_A V_N) = \frac{K_{IC}^{(A)}}{\sqrt{b_N f_N} \rho_A V_N} \quad (18)$$

in which

$$f_N = 2 \left(\frac{a_N}{b_N} \right)^{1/2} - 0.4 \left(\frac{a_N}{b_N} \right)^{3/2} + 18.7 \left(\frac{a_N}{b_N} \right)^{5/2} - 38.5 \left(\frac{a_N}{b_N} \right)^{7/2} + 53.9 \left(\frac{a_N}{b_N} \right)^{9/2}$$

$$g_N = 1 / (1 - a_N/b_N) (a_N/b_N < 0.6, N \geq 1)$$

whereas $\rho_0 = \rho_B$ and $E_0 = E_B$ are the corresponding density and elastic modulus of material B in the elementary level 1 respectively. $K_{IC}^{(A)}$ is the fracture toughness of material A and $K_{IC}^{(B)}$ is that of material B in the elementary level 1.

Here we simply suppose that the fracture toughness $K_{IC}^{(B)}|_N$ of material B at level N ($N \geq 2$) is proportional to the maximum relative strength-to-weight ratio derived at the previous level:

$$K_{IC}^{(B)}|_N = \begin{cases} K_{IC}^{(B)} & N = 1 \\ \frac{[(\sigma_R/W_R)|_{N-1}]_{\max}}{K_{IC}^{(B)}/K_{IC}^{(A)}} K_{IC}^{(B)}|_{N-1} & N \geq 2 \end{cases} \quad (19)$$

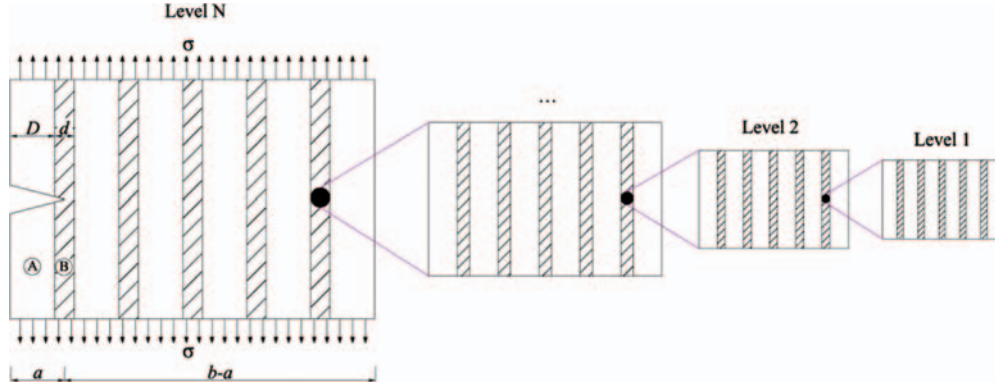


Fig. 5. The N -level hierarchical structures of the bi-material composites.

in which $(\sigma_R/W_R)|_N$ ($N \geq 1$) is the relative strength to weight ratio at level N ($N \geq 1$). Then $(\sigma_R/W_R)|_N$ can be analogically derived from Eqs. (15)–(17):

$$\begin{aligned} \frac{\sigma_R}{W_R} \Big|_N &= \frac{\sigma_N/W_N}{(\sigma_A/W_A)|_N} = \frac{\sigma_N/(\rho_N V_N)}{\sigma_A/(\rho_A V_N)} = g_N \frac{K_{IC}^B|_N}{K_{IC}^A} \\ &\times \frac{(n-1)E_A x + [(1-a_N/b_N)x + n(1-(a_N/b_N))]E_{N-1}}{nE_A x + nE_{N-1}} \frac{\rho_A}{\rho_N} \\ &= \begin{cases} \frac{K_{IC}^B|_N}{K_{IC}^A} g_N Y & N=1 \\ \prod_{i=2}^N \left[\frac{\sigma_R}{W_R} \Big|_{N-1} \right]_{\max} / \left(\frac{K_{IC}^{(B)}}{K_{IC}^{(A)}} \right)^{N-2} g_N Y & N \geq 2 \end{cases} \quad (20) \end{aligned}$$

in which

$$\begin{aligned} Y &= ((n-1)x(x+1)^{N-1} + [(1-(a_N/b_N))x + n(1-a_N/b_N)] \\ &\times [(x+1)^{N-1} - (1+E_B/E_A)] \cdot (nx(x+1)^{N-1} \\ &+ n[(x+1)^N - (1+E_B/E_A)])^{-1} \frac{(x+1)^N}{[(x+1)^N - 1] + \rho_B/\rho_A} \end{aligned}$$

An example is given in the following. Here we suppose $z = E_B/E_A = 5$, $y = \rho_B/\rho_A = 5$, $a_N/b_N = 0.01$ ($N = 1-6$)

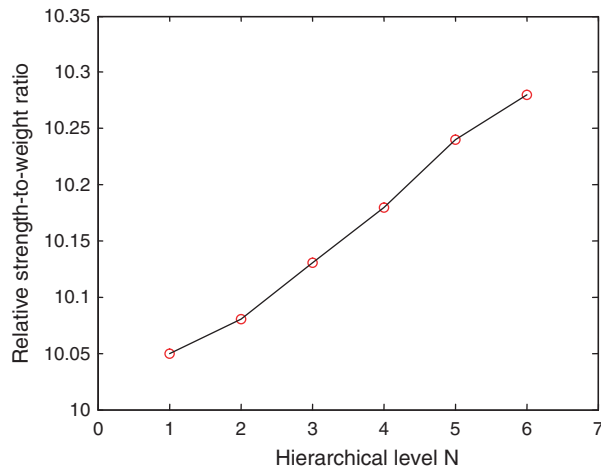


Fig. 6. The relative strength-to-weight ratio versus the number of the hierarchical level for the bi-material composites from level 1 to level 6.

and $K_{IC}^{(B)}/K_{IC}^{(A)} = 10$; the trends of the relative strength-to-weight ratios $(\sigma_R/W_R)|_N$ from level 1 ($N = 1$) to level 6 ($N = 6$) are shown in Figure 6. From Figure 6 we can see that when the crack spreads into the tougher reinforced material B the relative strength-to-weight ratio is proportional to $K_{IC}^{(B)}/K_{IC}^{(A)}$ and the maximum relative strength-to-weight ratio $(\sigma_R/W_R)|_N$ for the hierarchical bi-material composites is increasing with N . It means that in the practical design we can increase the strength-to-weight ratio of the bi-material composites by increasing the number of hierarchical levels optimizing the ratio $x = D/d$ at each hierarchical level, mimicking nature.

4. CONCLUSIONS

In this paper, the strength-to-weight ratio of a periodically distributed bi-material plate with hierarchy has been analyzed. The results show that the strength-to-weight ratio of the bi-material composites could be increased by increasing the number of hierarchical levels and optimized through choosing the appropriate width ratio of the two materials, suggesting a new bio-inspired material design.

APPENDIX

Some textile structures have the form of cross section showed in Figure 7(a) where the thicker sections correspond to crack arrests.²³⁻²⁵ We compare it with the bi-material plate showed in Figure 7(b). Suppose Figures 7(a) and (b) are equivalent and the heights of the two structures are the same, then $\rho_A H d = \rho_B h d$, $E_A H d = E_B h d$, i.e., $y = \rho_B/\rho_A = H/h = E_B/E_A = z$. Substituting $y = z$ into Eq. (11), it is easy to prove that when $y = z = 1$ the relative strength-to-weight ratio is $\sigma_R/W_R = 1$; when $y = z > 1$, σ_R/W_R is increasing versus $x = D/d$ and tends to 1; whereas when, $y = z \leq 1$, σ_R/W_R is decreasing versus $x = D/d$ and again tends to 1. Thus $H/h = y = z < 1$ is the one we need in order to optimize the strength-to-weight of the tissue structures. For example, suppose $H/h = y = z = 0.5$, when $x = D/d = 1$ $\sigma_R/W_R = 1.36$, the

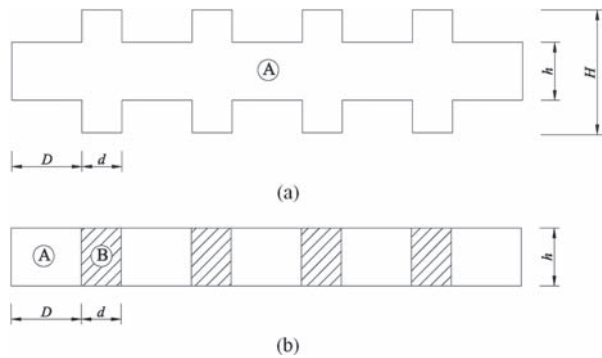


Fig. 7. Cross sections of (a) the tissue with crack arrests and (b) the bi-material composite analog.



Fig. 8. One optimized scheme for the tissue structures shown in Figure 7(a).

strength-to-weight ratio, the optimized tissue is showed in Figure 8.

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