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Erratum

A. Konstantinidis, P. Cornetti, N. Pugno and E.C. Aifantis, Application of Gradient Theory and Quantized Fracture Mechanics in Snow Avalanches, *J. Mech. Behav. Mater*. 19, 39–47, 2009

Nicola Pugno[1,*], Avraam Konstantinidis[2], Pietro Cornetti[1] and Elias C. Aifantis[2–4,*]

[1] Department of Structural Engineering, Politecnico di Torino, Corso Duca Degli Abruzzi 24, 10129 Torino, Italy, e-mail: nicola.pugno@polito.it
[2] Laboratory of Mechanics and Materials, Polytechnic School, Aristotle University of Thessaloniki, 54124, Thessaloniki, Greece, e-mail: mom@mom.gen.auth.gr
[3] Michigan Technological University, Houghton, MI 49931, USA
[4] King Abdulaziz University, Jeddah 21589, Saudi Arabia

* Corresponding authors

The authors would like to make some corrections/revisions to their previous article cited above. These pertain mainly to the way the introduction to the Quantized Fracture Mechanics method, which was initially proposed by the first author, should have appeared in order to be more clear and transparent for the benefit of the readers. In addition, an error in the form of the parameter α is corrected, thus leading to slightly different results. It turns out that this correction establishes closer connection between the two methods being compared, i.e., gradient theory and quantized fracture mechanics. This Erratum was due much earlier but due to misunderstandings with the previous JMBM publisher its appearance has been delayed. Below, detailed accounts of the revisions needed are indicated in bullets.

- **Introduction to Section "2. Theoretical Considerations" and Sub-section "2.1 QFM Formulation" (contained in p. 40–42) should be replaced as follows:**

We consider a snow slab of height h and width w adhering with shear stress τ_{EXT} to a snow weak layer of thickness t<<h forming an angle θ with respect to the horizontal plane (see Figure 1). The weak layer may form under specific environmental conditions. It is usually made of large crystals, which show a very low shear strength. The presence of such weak layer favours avalanche triggering; it represents the interface between the snow slab and the bedrock (or another, stiffer, snow layer). To satisfy equilibrium, the shear stress acting in the weak layer should be of the form:

$$\tau_{EXT}=\rho g h \sin\theta \tag{1}$$

[2]Professor, [3]Emeritus, [4]Adjunct

where ρ is the snow density and g is the gravity constant. If a defect of length 2a is present in the weak layer (the so-called super-weak zone), an axial force N(x) will occur in the debonded portion of the snow slab:

$$N(x)=w\int_0^x(\tau_{EXT}-\tau_\infty)dx=(\tau_{EXT}-\tau_\infty)wx \tag{2}$$

where τ_∞ is the residual shear strength after failure. Note that the upper part of the snow slab is in tension, whereas the lower one is compressed. Half of the strain energy stored in the (debonded part of) snow slab is therefore:

$$\Phi(a)=\frac{1}{2}\int_0^a\frac{N^2}{E'hw}dx=\frac{wa^3(\tau_{EXT}-\tau_\infty)^2}{6E'h} \tag{3}$$

where E′ is the Young modulus of the snow slab in plane strain conditions, i.e., $E'=E/(1-\nu^2)$.

2.1. QFM Formulation

Quantized Fracture Mechanics (QFM) is a recent energy-based theory firstly proposed by the first author and his co-workers [10]. It involves a quantization of Griffith's criterion to account for discrete crack propagation, thus in the continuum hypothesis, differentials are substituted with finite differences, i.e., d→Δ. According to the principle of energy conservation, Griffith's energy criterion implies that delamination will take place when the strain energy release rate G attains a value equal to the critical value G_C, i.e., the fracture energy:

$$G=\frac{d\Phi}{dS}=G_c \tag{4}$$

where S is the fracture surface. In the case of QFM, Eq. (4) takes the form:

$$G^*=\frac{\Delta\Phi}{\Delta S}=G_c \tag{5}$$

where ΔS=w×Δa; Δa represents the discrete crack length increment and should be regarded as a material property. Criterion (5) together with Eq. (3) yield:

$$G^*=\frac{\Phi(a+\Delta a)-\Phi(a)}{w\Delta a}=\frac{(\tau_{EXT}-\tau_\infty)^2}{6E'h}(3a^2+3a\,\Delta a+\Delta a^2)=G_c. \tag{6}$$

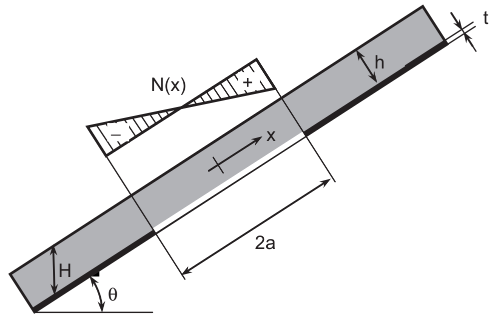

Figure 1 Geometry of the problem: snow slab of height h, weak layer of thickness t and super-weak zone of length 2a. H is the height of the fallen snow and N(x) the axial force in the debonded snow slab.

Eq. (6) provides the critical value τ_c of the external shear stress τ_{EXT}:

$$\tau_c = \tau_\infty + \sqrt{\frac{6E'hG_c}{3a^2 + 3a\,\Delta a + \Delta a^2}}. \tag{7}$$

It is interesting to observe that, differently from LEFM, the QFM criterion provides a finite strength also for a vanishing defect (i.e., $a \to 0$). This is one of the main advantage of using QFM instead of LEFM. In such a case, we then have:

$$\tau_c = \tau_\infty + \frac{\sqrt{6E'hG_c}}{\Delta a}, \tag{8}$$

which represents the shear strength in the absence of super-weak zones. On the other hand, for large basal defects (i.e., $a >> \Delta a$), the QFM provides the same result as the LEFM:

$$\tau_c = \tau_\infty + \frac{\sqrt{2E'hG_c}}{a} \approx \tau_\infty + \frac{\sqrt{2.17\,EhG_c}}{a}, \tag{9}$$

where the last equality holds for a Poisson's ratio of the snow equal to 0.2 [7]. Neglecting the residual shear strength, Eqs. (1) and (9) provide the critical height H_C (i.e., the fallen snow, $H = h/\cos\theta$) for the avalanche formation according to QFM as:

$$H_c^{QFM} \approx 2.17 \frac{EG_c}{(\rho\, g\, a \sin\theta)^2 \cos\theta}. \tag{10}$$

- **In p. 42, in the line after Eq. (11)**

"… and $\alpha=(3\lambda+2\mu)/3\mu=(2+2\nu)/3\mu$…" should read "… and $\alpha=(3\lambda+2\mu)/3\mu=(2/3)(1+\nu)/(1-2\nu)$…"Accordingly, the value "6.6" in Eqs. (24)-(25) must be replaced by "2.2".

- **In p. 43, in the line before Eq. (14)**

"… configuration is equated to the elastic energy, i.e.," should read "… configuration, i.e.,"

- **In p. 44, in the line before Eq. (19)**

"… $-\alpha \le x \le \alpha$…" should read "… $-a \le x \le a$ …"

- **In p. 45, in the line before Eq. (23)**

"…Setting $(\tau_{EXT}-\tau\infty) \equiv \tau$, Eq. (22) becomes" should read "…Hence Eq. (22) becomes"

- **In p. 45, Eq. (23) should read**

$$G_c = \frac{a^2(\tau_{EXT}-\tau_\infty)^2}{4\,\mu\, h\, \alpha} \quad \Rightarrow \quad \tau_c = \tau_\infty + \frac{\sqrt{4\, G_c\, \mu\, h\, \alpha}}{a}. \tag{23}$$

- **In p. 45, in the line before Eq. (24)**

"… parameter $\alpha=(2+2\nu)/(1-2\nu)$…" should read "… parameter $\alpha=(2/3)(1+\nu)/(1-2\nu)$…"

- **In p. 45, Eq. (24) should read**

$$\tau_c = \tau_\infty + \frac{\sqrt{2.22\,EhG_c}}{a} \tag{24}$$

- **In p. 45, in the line before Eq. (25)**

"… we obtain:" should read "… we obtain, neglecting the residual shear strength:"

- **In p. 45, Eq. (25) should read**

$$H_c^{grad} \approx 2.22 \frac{EG_c}{(\rho\, g\, a \sin\theta)^2 \cos\theta} \tag{25}$$

- **In p. 45, in the fourth line from the end of the page**

"… plot of the non-dimensional quantity $H_c\rho^2g^2/2G_cE$…" should read "… plot of the critical height H_c vs. slope θ"

- **In p. 45, in the second line from the end of the page**

"… slopes). As can be seen…" should read "… slopes). The snow properties are taken from [7]: E=1 MPa, ρ=200 kg/m^3 and G_c=0.2 J/m^2. As it can be seen…"

- **In p. 46, Figure 3 should be replaced with the one below.** In this connection, Section 3 must be replaced by the following:

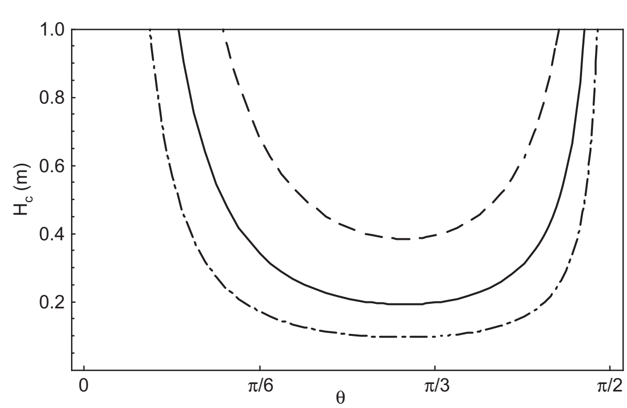

Figure 3 Predictions of the critical height of the fallen snow for avalanche triggering vs. slope. Different curves refer to different initial lengths (2a) of the super-weak zone: 1.5 m (dashed line); 3 m (continuous line) and 6 m (dash-dotted line).

As can be seen by Eqs. (10) and (25) both the QFM and gradient models give quite similar predictions for the critical height of the fallen show for avalanche triggering. Differentiation of both expressions for the critical fallen snow height provides the value for the critical slope (i.e., the slope for which avalanche triggering is easier) of about 54°, and two vertical asymptotes for $\theta=0°$ and $\theta=90°$, both reasonable. Figure 3 shows the plot of the critical height H_c vs. slope θ given by Eq. (10) for different values of the crack length a (note that the gradient model predictions, given by Eq. (25), have the same trends but with somewhat different slopes). The snow properties are taken from [7]: E=1 MPa, $\rho=200$ kg/m^3 and $G_c=0.2$ J/m^2. As it can be seen from Figure 3, the larger the interfacial crack, the smaller the value of the critical height of the fallen snow is.

- **In p. 47, Reference 7 should read**

7. B. Chiaia, P. Cornetti, B. Frigo. Triggering of dry snow slab avalanches: stress versus fracture mechanical approach. *Cold Regions Science and Technology*, **53**, 170–178 (2008).