

Application of Gradient Theory and Quantized Fracture Mechanics in Snow Avalanches

A. Konstantinidis¹, P. Cornetti², N. Pugno² and E.C. Aifantis^{1,3}

¹*Laboratory of Mechanics and Materials, Polytechnic School, Aristotle University of Thessaloniki, 54124, Thessaloniki, Greece*

²*Department of Structural Engineering, Politecnico di Torino, Corso Duca Degli Abruzzi 24, 10129 Torino, Italy*

³*Center for the Mechanics of Material Instabilities and Manufacturing Processes, Michigan Technological University, Houghton, MI 49931, USA*

ABSTRACT

Avalanche initiation in snow slabs of high enough length-to thickness ratio by failure of the interface between the snow slab and the underlying bedrock is considered. In this direction Quantized Fracture Mechanics (QFM), an extension of the theory of Linear Elastic Fracture Mechanics (LEFM), as well as gradient theory are employed. Mode-II fracture is assumed in both cases. The two models, although having a completely different origin, lead to similar expressions for the height of the fallen snow, which is critical for slab avalanche triggering.

1. INTRODUCTION

In this study snow avalanche triggering is studied by assuming a pre-existing shear crack at the interface between the snow slab and the bedrock, in analogy with the work of Palmer and Rice /1/ on shear bands in over-consolidated clay, and McClung /2/ in snow slab avalanches. Analogous work has been conducted by Fyffe *et al* /3/, Zaiser *et al* /4/ and Konstantinidis *et al* /5/ where, in addition, variations in interface toughness due to the presence of small-scale heterogeneities were considered. In particular, Konstantinidis *et al* /5/ employed a Linear Elastic Fracture Mechanics (LEFM) approach (Pugno and Carpinteri /6/; Chiaia *et al* /7/), as well as a gradient model, similar to the ones used successfully in metal plasticity (Aifantis /8/; Zbib and Aifantis /9/), in order to evaluate a critical value of the energy release rate G for avalanche initiation, thus

obtaining critical value for the height of the fallen snow. In this work an enhancement of LEFM, the Quantized Fracture Mechanics (QFM) formulation which was first proposed by the third author and his co-workers /10/ is employed and its predictions are compared with the ones coming from the gradient model. A brief description of the employed theoretical models is provided and Section 2, while in Section 3 a comparison of the relevant theoretical predictions is made.

2. THEORETICAL CONSIDERATIONS

We consider a snow slab of height h , length L and width w adhering with shear stresses τ to an oblique substrate forming an angle θ with respect to the horizontal plane, as shown in Figure 1, as well as a weak layer of thickness $t \ll h$ at the interface between the snow slab and the bedrock. It is assumed that the snow has a density ρ in a gravity field of acceleration g , so that an axial force $N(x)$ is present at the generic cross-section x .

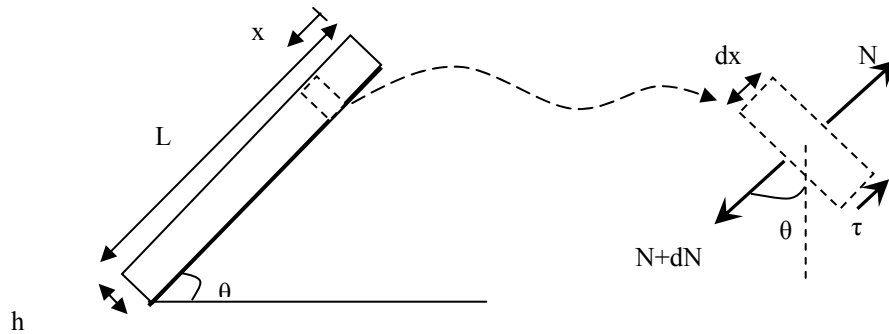


Fig. 1: Geometry of the problem

Equilibrium of forces of an infinitesimal element with length dx of the snow layer requires that

$$F_w + F_{ext} - F_f = 0 \Rightarrow \rho g w h \sin \theta dx + \frac{N + dN - N}{dx} dx - \tau w dx = 0 \Rightarrow \rho g \sin \theta + \frac{d\sigma}{dx} - \frac{\tau}{h} = 0 \quad (1)$$

where F_w , F_{ext} , F_f are the gravitational force, the axial force at the cross section and the adhesion force, respectively. The normal stress in the snow is given by $\sigma = N/(hw)$.

We next assume a linear elastic constitutive law for the adhesion stress τ , as

$$\tau = G_i \gamma = G_i \frac{u}{t} \quad (2)$$

where G_i is the interface shear modulus, γ the shear strain and u the (elastic plus rigid) displacement of the snow. The compatibility equation implies:

$$u = \frac{1}{E} \int_0^x \sigma \, dx + u_0 = \frac{t \tau}{G_i} \quad (3)$$

where E denotes the Young's modulus of the snow and u_0 is a constant, representing the rigid displacement. Accordingly, the equilibrium equation becomes:

$$\frac{d^2 \sigma}{dx^2} - \frac{G_i}{htE} \sigma = 0, \quad (4)$$

which by integration and assuming zero normal stress σ at the upper and lower end of the slope, i.e. $\sigma|_{x=0} = \sigma|_{x=L} = 0$, leads to

$$\sigma = 0 \quad ; \quad \tau = \rho gh \sin \theta. \quad (5)$$

It should be noted that the model allows us to consider without any problems different boundary conditions, e.g. a weight acting on the upper part of the snow due to an interface crack. For a detailed discussion of this model see /7/.

2.1 QFM Formulation

Quantized Fracture Mechanics (QFM) is a recent energy-based theory first proposed by the third author and his co-workers /10/. It involves a quantization of Griffith's criterion to account for discrete crack propagation, thus in the continuum hypothesis, differentials are substituted with finite differences, i.e. $d \rightarrow \Delta$. According to the principle of energy conservation, Griffith's energy criterion implies that delamination will take place when the energy release rate G (the opposite of the variation of the total potential energy W with respect to the crack surface S) attains a value equal to a critical value G_C , i.e.

$$G = -\frac{dW}{dS} = G_C. \quad (6)$$

In the case of QFM the Eq. (6) takes the form

$$G^* = -\frac{\Delta W}{\Delta S} = G_C. \quad (7)$$

Applying Clapeyron theorem and neglecting the friction work, $\Delta W = -\Delta H$ with H denoting the elastic energy /6/. For QFM the variation of elastic energy due to the debonding of a length a (the variation of the elastic energy stored in the interface can be neglected, due to its small volume, as can be easily demonstrated) is given by

$$\Delta H = \int_0^a \frac{1}{2} \frac{\rho^2 g^2 (hw)^2 \sin^2 \theta}{Ehw} x^2 dx = \frac{1}{6} \frac{\rho^2 g^2 hw \sin^2 \theta}{E} a^3 = \frac{1}{6} \frac{\tau^2 w}{Eh} a^3 \quad (8)$$

Combining Eqs. (6) - (8) provide the critical nominal tangential stress as

$$G = -\frac{\Delta W}{\Delta S} = \frac{\Delta H}{wa} = \frac{\tau^2 a^2}{6Eh} \Rightarrow \tau_c^2 = \frac{6Eh_c G^*}{a^2} \Rightarrow \tau_c = \frac{\sqrt{6Eh_c G^*}}{a}. \quad (9)$$

In order to get the critical height H_C (i.e. the fallen snow, $H = h/\cos \theta$) for the avalanche formation, we have:

$$\tau_c = \rho g h \sin \theta = \frac{\sqrt{6EG_C h}}{a} \Rightarrow H_C^{\text{QFM}} = \frac{6G_C E}{\rho^2 g^2 \cos \theta \sin^2 \theta a^2}. \quad (10)$$

2.2 Gradient Formulation

In order to analyze the propagation of a crack between the snow slab and the underlying bedrock the elasticity problem is solved /4/. The elastic energy functional associated with a general displacement vector field $\mathbf{w}(\mathbf{r})$ in an isotropic material has the form

$$H(\mathbf{w}) = \frac{\mu}{2} \int \left[\alpha (\text{div} \mathbf{w})^2 + (\text{curl} \mathbf{w})^2 \right] d^3 r \quad (11)$$

where ν is the Poisson's ratio and $\alpha = (3\lambda + 2\mu)/3\mu = (2 + 2\nu)/(1 - 2\nu)$. The associated equilibrium equation reads

$$\nabla^2 \mathbf{w} + \frac{1}{1-2\nu} \text{grad}(\text{div} \mathbf{w}) = 0, \quad (12)$$

and can be solved in the Fourier space by assuming that the height h of the slab is much smaller than the characteristic length of variations in the displacement field using $w_x(x, y, 0) = u(x, y)$, $w_y = w_z = 0$. The energy functional then becomes

$$H(u) = h\mu \int \int \left[\alpha (\partial_x u)^2 + (\partial_y u)^2 \right] dx dy \quad (13)$$

and is equated to the work that has to be expended against the shear stress $\tau_{\text{int}}(x, y)$ in order to create the displacement field $u(x, y)$ from an initially displacement-free configuration is equated to the elastic energy, i.e.

$$H(u) = \int \int \left[\int_0^{u(x,y)} \tau_{\text{int}} du \right] dx dy . \quad (14)$$

Taking the functional derivative with respect to $u(x, y)$ on both sides of Eq. (14), the internal stress for the 2-D case is given by

$$\tau_{\text{int}} = c_{\text{II}} \partial_x^2 u + c_{\text{III}} \partial_y^2 u , \quad (15)$$

where the so-called gradient coefficients are $c_{\text{II}} = 2h\mu\alpha$, $c_{\text{III}} = 2h\mu$. For the 1-D case, we consider mode-II fracture (u is a function of the x coordinate only), thus Eq. (15) reduces to

$$\tau_{\text{int}} = c_{\text{II}} \partial_x^2 u \quad (16)$$

We further assume that the interface shear strength vs. shear displacement curve shows a hardening-softening form as shown schematically in Figure 2, with a shear strength increase towards a peak value τ_M and then a drop towards an asymptotic value τ_∞ .

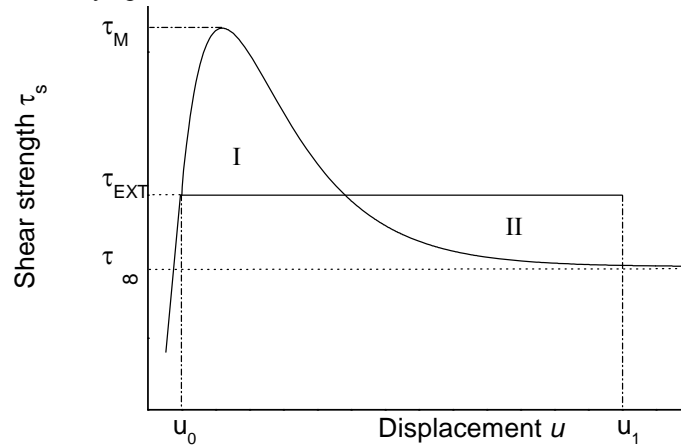


Fig. 2: Shear strength versus displacement across the interface.

Considering a slope where a critical (marginally stable) mode-II crack exists along the interface between the snow and the bedrock, the displacement satisfies the equation

$$c_{II}u_{xx} + \tau_{EXT} - \tau_S(u, x) = 0 \quad (17)$$

where $\tau_S(u, x)$ is the shear strength. Equations of this type have been studied by Aifantis and co-workers in the different context of shear and slip bands in metal plasticity (Aifantis /7/; Zbib and Aifantis /8/) containing a strain variable (shear strain or equivalent strain) in place of the displacement variable u . The mathematical structure is, however, the same as in the present problem.

Equation (17) can be envisaged as describing the un-damped motion of a particle of mass c_{II} in a potential. It follows that the solution must satisfy the 'energy conservation' criterion

$$\int_{u_0}^{u_1} (\tau_{EXT} - \tau_S(u)) du = 0 . \quad (18)$$

If the length of the crack is large in comparison with that of the end region over which the strength of the weak layer drops to the residual value τ_∞ , approximate analytical relations can be derived /4/. By integrating Eq. (18), the displacement profile for $-\alpha \leq x \leq \alpha$ is approximately given by

$$u(x) = \frac{(a^2 - x^2)(\tau_{EXT} - \tau_\infty)}{2c_{II}} \quad (19)$$

and the maximum displacement (at the origin) is $u_1 = a^2(\tau_{EXT} - \tau_\infty)/2c_{II}$. As in /4/, area II in Figure 2 can be approximated by $u_1(\tau_{EXT} - \tau_\infty)$, and area I can because of the smallness of $(\tau_{EXT} - \tau_\infty)$ be approximated by

$$\int_{u_0}^{u_1^*} [\tau_{EXT} - \tau_S(u)] du \approx \int_{u_0}^{\infty} [\tau_{EXT} - \tau_S(u)] du . \quad (20)$$

Hence, the equal-area condition, Eq. (18), can be written in the approximate form

$$u_1(\tau_{EXT} - \tau_\infty) = (\tau_M - \tau_\infty)\bar{u} \quad \text{where} \quad (\tau_M - \tau_\infty)\bar{u} := \int (\tau_S(u) - \tau_\infty) du . \quad (21)$$

Using Eqs. (19) - (21), we finally obtain the approximate relationship

$$G_C = \frac{a^2(\tau_{EXT} - \tau_\infty)^2}{2c_{II}} = (\tau_M - \tau_\infty)\bar{u} \quad (22)$$

for a marginally stable crack. Equation (22) may be interpreted as a Griffith-like energy balance criterion. The term on the right-hand side can be interpreted as the interface toughness; it represents the work per unit crack length which has to be done in order to reduce the strength of the interface as the crack advances by a unit amount. The left-hand side corresponds to the elastic energy released per unit crack length as the crack advances by a unit amount, i.e., it is the effective force per unit length acting on the crack. Crack propagation occurs as soon as the elastic energy release exceeds the effective toughness. Setting $(\tau_{EXT} - \tau_{\infty}) \equiv \tau$, Eq. (22) becomes

$$G_C = \frac{a^2 \tau^2}{4\mu h \alpha} \Rightarrow \tau = \tau_c = \frac{\sqrt{4G_C h \mu \alpha}}{a}. \quad (23)$$

Substituting in Eq. 23 the shear modulus as $\mu = E/(2+2\nu)$ and parameter $\alpha = (2+2\nu)/(1-2\nu)$ and assuming for the Poisson's ratio of the snow the value 0.2 /7/, we have

$$\tau_c \approx \frac{\sqrt{6.6G_C E h_c}}{a} \quad (24)$$

Wishing to get the critical height H_C of the fallen snow, we obtain:

$$H_C^{\text{grad}} = \frac{6.6 G_C E}{\rho^2 g^2 \cos^2 \theta \sin^2 \theta a^2} \quad (25)$$

3. COMPARISON BETWEEN THE QFM AND GRADIENT MODELS PREDICTIONS

As can be seen from Eqs. (10) and (25), both the QFM and gradient models give quite similar predictions for the critical height of the fallen snow for avalanche triggering. Differentiation of both expressions for the critical fallen snow height provides the value for the critical slope (i.e. the slope for which avalanche triggering is easier) of about 54° , and two vertical asymptotes for $\theta = 0^\circ$ and $\theta = 90^\circ$, both reasonable. Figure 3 shows the plot of the non-dimensional quantity $H_C \rho^2 g^2 / 2G_C E$ given by Eq. (10) for different values of the crack length a (note that the gradient model predictions, given by Eq. (25), have the same trends but with somewhat different slopes). As can be seen from Figure 3, the larger the interfacial crack, the smaller the value of the critical height of the fallen snow is.

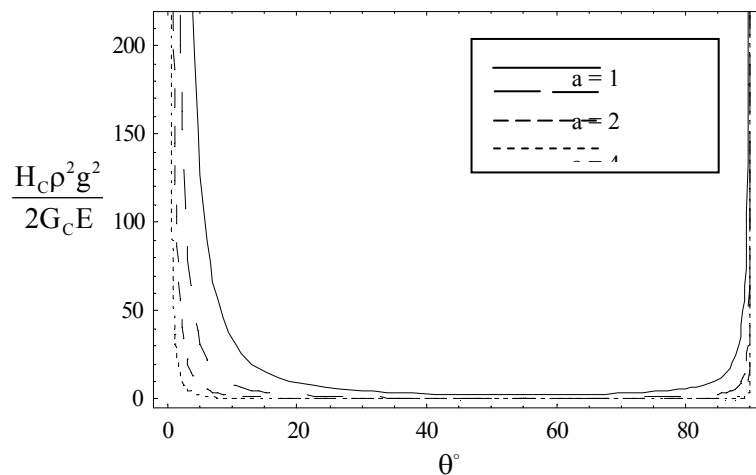


Fig. 3: Predictions for the critical height of the fallen snow for avalanche triggering of the two models.

ACKNOWLEDGMENTS

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Erratum

A. Konstantinidis, P. Cornetti, N. Pugno and E.C. Aifantis, Application of Gradient Theory and Quantized Fracture Mechanics in Snow Avalanches, *J. Mech. Behav. Mater.* 19, 39–47, 2009

Nicola Pugno^{1,*}, Avraam Konstantinidis², Pietro Cornetti¹ and Elias C. Aifantis^{2-4,*}

¹Department of Structural Engineering, Politecnico di Torino, Corso Duca Degli Abruzzi 24, 10129 Torino, Italy, e-mail: nicola.pugno@polito.it

²Laboratory of Mechanics and Materials, Polytechnic School, Aristotle University of Thessaloniki, 54124, Thessaloniki, Greece, e-mail: mom@mom.gen.auth.gr

³Michigan Technological University, Houghton, MI 49931, USA

⁴King Abdulaziz University, Jeddah 21589, Saudi Arabia

*Corresponding authors

The authors would like to make some corrections/revisions to their previous article cited above. These pertain mainly to the way the introduction to the Quantized Fracture Mechanics method, which was initially proposed by the first author, should have appeared in order to be more clear and transparent for the benefit of the readers. In addition, an error in the form of the parameter α is corrected, thus leading to slightly different results. It turns out that this correction establishes closer connection between the two methods being compared, i.e., gradient theory and quantized fracture mechanics. This Erratum was due much earlier but due to misunderstandings with the previous JMBM publisher its appearance has been delayed. Below, detailed accounts of the revisions needed are indicated in bullets.

- **Introduction to Section “2. Theoretical Considerations” and Sub-section “2.1 QFM Formulation” (contained in p. 40–42) should be replaced as follows:**

We consider a snow slab of height h and width w adhering with shear stress τ_{EXT} to a snow weak layer of thickness $t \ll h$ forming an angle θ with respect to the horizontal plane (see Figure 1). The weak layer may form under specific environmental conditions. It is usually made of large crystals, which show a very low shear strength. The presence of such weak layer favours avalanche triggering; it represents the interface between the snow slab and the bedrock (or another, stiffer, snow layer). To satisfy equilibrium, the shear stress acting in the weak layer should be of the form:

$$\tau_{EXT} = \rho gh \sin\theta \tag{1}$$

²Professor, ³Emeritus, ⁴Adjunct

where ρ is the snow density and g is the gravity constant. If a defect of length $2a$ is present in the weak layer (the so-called super-weak zone), an axial force $N(x)$ will occur in the debonded portion of the snow slab:

$$N(x) = w \int_0^x (\tau_{EXT} - \tau_{\infty}) dx = (\tau_{EXT} - \tau_{\infty}) wx \tag{2}$$

where τ_{∞} is the residual shear strength after failure. Note that the upper part of the snow slab is in tension, whereas the lower one is compressed. Half of the strain energy stored in the (debonded part of) snow slab is therefore:

$$\Phi(a) = \frac{1}{2} \int_0^a \frac{N^2}{E'hw} dx = \frac{wa^3 (\tau_{EXT} - \tau_{\infty})^2}{6E'h} \tag{3}$$

where E' is the Young modulus of the snow slab in plane strain conditions, i.e., $E' = E/(1-\nu^2)$.

2.1. QFM Formulation

Quantized Fracture Mechanics (QFM) is a recent energy-based theory firstly proposed by the first author and his co-workers [10]. It involves a quantization of Griffith’s criterion to account for discrete crack propagation, thus in the continuum hypothesis, differentials are substituted with finite differences, i.e., $d \rightarrow \Delta$. According to the principle of energy conservation, Griffith’s energy criterion implies that delamination will take place when the strain energy release rate G attains a value equal to the critical value G_c , i.e., the fracture energy:

$$G = \frac{d\Phi}{dS} = G_c \tag{4}$$

where S is the fracture surface. In the case of QFM, Eq. (4) takes the form:

$$G^* = \frac{\Delta\Phi}{\Delta S} = G_c \tag{5}$$

where $\Delta S = w \times \Delta a$; Δa represents the discrete crack length increment and should be regarded as a material property. Criterion (5) together with Eq. (3) yield:

$$G^* = \frac{\Phi(a+\Delta a) - \Phi(a)}{w\Delta a} = \frac{(\tau_{EXT} - \tau_{\infty})^2}{6E'h} (3a^2 + 3a\Delta a + \Delta a^2) = G_c \tag{6}$$

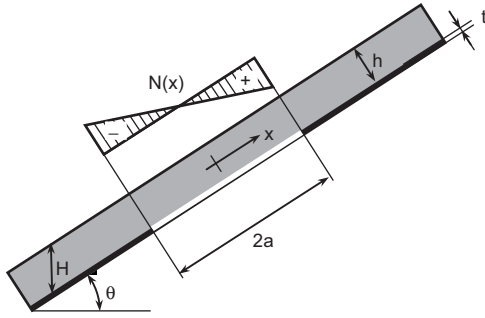


Figure 1 Geometry of the problem: snow slab of height h , weak layer of thickness t and super-weak zone of length $2a$. H is the height of the fallen snow and $N(x)$ the axial force in the debonded snow slab.

Eq. (6) provides the critical value τ_c of the external shear stress τ_{EXT} :

$$\tau_c = \tau_\infty + \sqrt{\frac{6E'hG_c}{3a^2 + 3a\Delta a + \Delta a^2}}. \quad (7)$$

It is interesting to observe that, differently from LEFM, the QFM criterion provides a finite strength also for a vanishing defect (i.e., $a \rightarrow 0$). This is one of the main advantage of using QFM instead of LEFM. In such a case, we then have:

$$\tau_c = \tau_\infty + \frac{\sqrt{6E'hG_c}}{\Delta a}, \quad (8)$$

which represents the shear strength in the absence of super-weak zones. On the other hand, for large basal defects (i.e., $a \gg \Delta a$), the QFM provides the same result as the LEFM:

$$\tau_c = \tau_\infty + \frac{\sqrt{2E'hG_c}}{a} \approx \tau_\infty + \frac{\sqrt{2.17 EhG_c}}{a}, \quad (9)$$

where the last equality holds for a Poisson's ratio of the snow equal to 0.2 [7]. Neglecting the residual shear strength, Eqs. (1) and (9) provide the critical height H_c (i.e., the fallen snow, $H = h/\cos \theta$) for the avalanche formation according to QFM as:

$$H_c^{QFM} \approx 2.17 \frac{EG_c}{(\rho g a \sin \theta)^2 \cos \theta}. \quad (10)$$

• **In p. 42, in the line after Eq. (11)**

"... and $\alpha = (3\lambda + 2\mu)/3\mu = (2+2\nu)/3\mu$..." should read "... and $\alpha = (3\lambda + 2\mu)/3\mu = (2/3)(1+\nu)/(1-2\nu)$..." Accordingly, the value "6.6" in Eqs. (24)-(25) must be replaced by "2.2".

• **In p. 43, in the line before Eq. (14)**

"... configuration is equated to the elastic energy, i.e.," should read "... configuration, i.e.,"

• **In p. 44, in the line before Eq. (19)**

"... $-\alpha \leq x \leq \alpha$..." should read "... $-a \leq x \leq a$..."

• **In p. 45, in the line before Eq. (23)**

"...Setting $(\tau_{EXT} - \tau_\infty) \equiv \tau$, Eq. (22) becomes" should read "...Hence Eq. (22) becomes"

• **In p. 45, Eq. (23) should read**

$$G_c = \frac{a^2 (\tau_{EXT} - \tau_\infty)^2}{4 \mu h \alpha} \Rightarrow \tau_c = \tau_\infty + \frac{\sqrt{4 G_c \mu h \alpha}}{a}. \quad (23)$$

• **In p. 45, in the line before Eq. (24)**

"... parameter $\alpha = (2+2\nu)/(1-2\nu)$..." should read "... parameter $\alpha = (2/3)(1+\nu)/(1-2\nu)$..."

• **In p. 45, Eq. (24) should read**

$$\tau_c = \tau_\infty + \frac{\sqrt{2.22 EhG_c}}{a} \quad (24)$$

• **In p. 45, in the line before Eq. (25)**

"... we obtain:" should read "... we obtain, neglecting the residual shear strength:"

• **In p. 45, Eq. (25) should read**

$$H_c^{grad} \approx 2.22 \frac{EG_c}{(\rho g a \sin \theta)^2 \cos \theta} \quad (25)$$

• **In p. 45, in the fourth line from the end of the page**

"... plot of the non-dimensional quantity $H_c \rho^2 g^2 / 2G_c E$..." should read "... plot of the critical height H_c vs. slope θ "

• **In p. 45, in the second line from the end of the page**

"... slopes). As can be seen..." should read "... slopes). The snow properties are taken from [7]: $E = 1$ MPa, $\rho = 200$ kg/m³ and $G_c = 0.2$ J/m². As it can be seen..."

• **In p. 46, Figure 3 should be replaced with the one below.** In this connection, Section 3 must be replaced by the following:

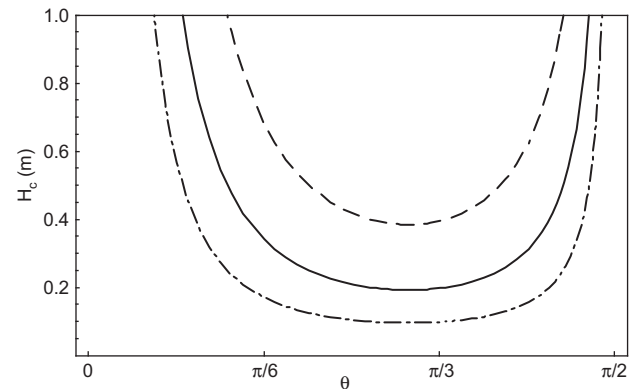


Figure 3 Predictions of the critical height of the fallen snow for avalanche triggering vs. slope. Different curves refer to different initial lengths ($2a$) of the super-weak zone: 1.5 m (dashed line); 3 m (continuous line) and 6 m (dash-dotted line).

As can be seen by Eqs. (10) and (25) both the QFM and gradient models give quite similar predictions for the critical height of the fallen snow for avalanche triggering. Differentiation of both expressions for the critical fallen snow height provides the value for the critical slope (i.e., the slope for which avalanche triggering is easier) of about 54° , and two vertical asymptotes for $\theta=0^\circ$ and $\theta=90^\circ$, both reasonable. Figure 3 shows the plot of the critical height H_c vs. slope θ given by Eq. (10) for different values of the crack length a (note that the gradient model predictions, given by Eq. (25), have the

same trends but with somewhat different slopes). The snow properties are taken from [7]: $E=1$ MPa, $\rho=200$ kg/m³ and $G_c=0.2$ J/m². As it can be seen from Figure 3, the larger the interfacial crack, the smaller the value of the critical height of the fallen snow is.

• **In p. 47, Reference 7 should read**

7. B. Chiaia, P. Cornetti, B. Frigo. Triggering of dry snow slab avalanches: stress versus fracture mechanical approach. *Cold Regions Science and Technology*, **53**, 170–178 (2008).